

Math 481: Homework 5

Due Wednesday, March 11, 2020

1. Consider the function

$$f: \mathbb{R}\mathbb{P}^2 \rightarrow \mathbb{R}$$
$$[x^1, x^2, x^3] \mapsto \frac{3(x^1)^2 + (x^2)^2}{(x^1)^2 + (x^2)^2 + (x^3)^2}$$

- (a) Choose a chart whose domain contains the point $p = [1, 1, 0] \in \mathbb{R}\mathbb{P}^2$ and express $df(p)$ in the corresponding local coordinates.
- (b) Choose a different chart whose domain contains p . Use the change of basis formula

$$dx_U^j = \sum_{i=1}^n \frac{\partial x_U^j}{\partial x_V^i}(p) dx_V^i$$

to express $df(p)$ in these new coordinates.

- (c) Consider the curve $\gamma: (-1, 1) \rightarrow \mathbb{R}\mathbb{P}^2$, $\gamma(t) = [\cos t, 1, \sin t]$. Compute $df(p)(\dot{\gamma}(0))$.

2. Construct a basis for $\mathcal{F}_{2,2}(\mathbb{R}^2)$ and prove that it is a basis.