

Math 481: Homework 4

Due Friday, February 28, 2020

1. Solve the Initial Value Problems encoded by the following vector fields on \mathbb{R}^2 . Write your solution in the form of a map $\phi_t : \mathbb{R}^2 \rightarrow \mathbb{R}^2$.

(a) $V(x) = x^2 \frac{\partial}{\partial x^1} \Big|_x - x^1 \frac{\partial}{\partial x^2} \Big|_x$.

(b) $W(x) = (x^1)^2 \frac{\partial}{\partial x^1} \Big|_x + 3x^2 \frac{\partial}{\partial x^2} \Big|_x$. (Is the solution of W defined for all $t \in \mathbb{R}$?)

2. Prove that for any smooth manifold M the map $\pi : TM \rightarrow M$ is smooth.
3. Let M be a two-dimensional manifold. Suppose there are two vector fields on M , say V and W , such that $\text{Span}\{V(p), W(p)\} = T_p M$ for all $p \in M$. Construct a diffeomorphism from TM to $M \times \mathbb{R}^2$.