

## Math 481: Homework 3

Due Friday, February 21, 2020

1. (a) Use the Regular Value Theorem to prove that the set

$$\left\{ (x, y, z) \in \mathbb{R}^3 \mid \left( \sqrt{x^2 + y^2} - 2 \right)^2 + z^2 = 1 \right\}$$

is an embedded submanifold of  $\mathbb{R}^3$ .

- (b) Draw the set. *Hint:* Use cylindrical coordinates  $(r, \theta, z)$  on  $\mathbb{R}^3$ .

2. Consider the function

$$f: \mathbb{R}P^2 \rightarrow \mathbb{R}$$
$$[x^1, x^2, x^3] \mapsto \frac{3(x^1)^2}{(x^1)^2 + (x^2)^2 + (x^3)^2}$$

- (a) Find all of the critical points of  $f$ . *Hint:* You need to check in all three of our standard coordinate charts  $\{(U_i, \phi_i)\}_{i=1,2,3}$ .
- (b) Find the point at which  $f$  takes its maximum value.
- (c) Find all values  $c$  for which the Regular Value Theorem implies that  $f^{-1}(c)$  is a manifold.