Math 417: Homework 10

Due Friday, November 19, 2021

1. **(5 points)** Goodman, exercise 5.4.2.
2. **(5 points)** Goodman, exercise 5.4.3.
3. **(10 points)** Consider the group $G = \mathrm{SL}(2, \mathbb{Z}_3)$ consisting of 2×2 matrices with entries in $\mathbb{Z}_3 = \{[0], [1], [2]\}$ that have determinant equal to $[1]$. Find a 3-Sylow subgroup of G and write out its elements. Show that the subgroup generated by

$$\begin{pmatrix} [0] & [2] \\ [1] & [0] \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} [1] & [1] \\ [1] & [2] \end{pmatrix}$$

   is a 2-Sylow subgroup of G.
4. **(5 points)** Show that if n is an odd number, then D_{2n} is isomorphic to $D_n \times \mathbb{Z}_2$.
5. **(5 points)** Goodman, exercise 6.1.4.
6. **(5 points)** Goodman, exercise 6.1.6.
7. **(5 points)** Goodman, exercise 6.1.9.