

Math 417: Homework 9

Due Friday, November 12, 2021

1. Goodman, exercise 5.2.1.
2. Goodman, exercise 5.2.2.
3. Goodman, exercise 5.2.3. Before attempting this problem, please read Section 4.1 of the text-book, particularly pages 220–221, where the rotational symmetry group of the cube is explained.
4. Goodman, exercise 5.4.1. *Hint:* Use the result of exercise 2.7.11 (Problem 8 on Homework 6).
5. Goodman, exercise 5.4.5.
6. Goodman, exercise 5.4.6. The homomorphisms $\varphi_1, \varphi_2, \varphi_3$ are the ones given on page 261.
7. Goodman, exercise 5.4.8.
8. Prove that any group of order 20 is isomorphic to a semidirect product of one of the following forms:
 - (a) $\mathbb{Z}_5 \rtimes_{\alpha} \mathbb{Z}_4$ for some homomorphism $\alpha : \mathbb{Z}_4 \rightarrow \text{Aut}(\mathbb{Z}_5)$, or
 - (b) $\mathbb{Z}_5 \rtimes_{\beta} (\mathbb{Z}_2 \times \mathbb{Z}_2)$ for some homomorphism $\beta : \mathbb{Z}_2 \times \mathbb{Z}_2 \rightarrow \text{Aut}(\mathbb{Z}_5)$.