

Math 417: Homework 7

Due Friday, October 22, 2021

1. (5 points) Goodman, exercise 2.7.4.
2. (5 points) Let D_n be the group of symmetries of the regular n -gon (dihedral group). In the terminology of Section 2.3, D_n consists of n rotations and n flips. Let N be the subgroup consisting of the rotations. Prove that N is normal. Prove that D_n/N is a cyclic group of order 2.
3. (15 points) Let G be the following group of 2×2 matrices:

$$G = \left\{ \begin{bmatrix} x & z \\ 0 & y \end{bmatrix} : x, y, z \in \mathbb{R}, x \neq 0, y \neq 0 \right\}.$$

You may take for granted that this set is a group under matrix multiplication. Prove that there is a subgroup $H \leq G$ such that

- H is normal in G ,
 - H is abelian, and
 - G/H is abelian.
4. (5 points) Goodman, exercise 3.1.9.
 5. (5 points) Goodman, exercise 3.1.13.