Math 417: Homework 4

Due Friday, September 24, 2021

1. Goodman, exercise 2.1.8.
2. Goodman, exercise 2.1.9.
3. Goodman, exercise 2.1.10.
4. Goodman, exercise 2.1.11.
5. Goodman, exercise 2.2.6.
6. Let a be an element of a group such that $a^k = e$ for some $k > 0$. Let n be the least positive integer such that $a^n = e$. Show that the elements $e, a, a^2, \ldots, a^{n-1}$ are all distinct.

   Note: This is one of the steps in the used in the proof of the classfication of cyclic groups, but it was not proved in the text nor in the lecture notes. Since the goal of this exercise is to complete that proof, you may not use Proposition 2.2.17 or 2.2.20 in your solution.
7. In the permutation group S_n, suppose that σ_1 is a k-cycle, σ_2 is an ℓ-cycle, and σ_1 and σ_2 are disjoint cycles. Find the order of the product $\sigma_1\sigma_2$ in terms of k and ℓ, and justify your answer.