Lecture 18 Proving relations, II.

We have so far discussed how one may show $\partial\circ\partial = 0$ and $\partial\circ C\Phi_S = C\Phi_S\circ\partial$.

Next we claim that

(a) Φ_S is independent of the choice of complex structure and perturbation data (relative fixed Floer data on striplike ends.)

(b) The gluing axiom holds

(c) $HF^{pr}(L_0, L_1)$ is independent of choice of Floer data up to isomorphism.

For all of these we must use <u>parametrized moduli spaces</u> (A key ingredient of chain level structures such as the Fukaya category)

Let R be a manifold, possibly with boundary and corners.
Let $\mathcal{S} = \{S_r\}$ be a family of pointed boundary Riemann surfaces parametrized by $r\in R$ (complex structure varies.) Pick a set of Lagrangian labels, and let (K_r, J_r) be a family of perturbation data. Assume Floer data is fixed on the ends. Let

$$\mathcal{M}_{S_r}(\{y_s\}) := \{\text{solutions } u: S_r \to M \text{ for } (K_r, J_r)\}$$

and

$$\mathcal{M}_{\mathcal{S}}(\{y_s\}) = \bigcup_{r\in R} \mathcal{M}_{S_r}(\{y_s\})$$

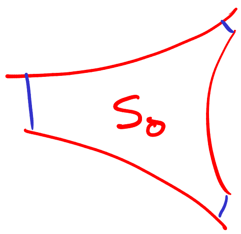
This is the parametrized moduli space for the family (S_r, K_r, J_r)

Under appropriate hypotheses, $\mathcal{M}_\delta(\{y_s\})$ can be given the structure of a manifold, and it admits a Gromov compactification $\overline{\mathcal{M}}_\delta(\{y_s\})$.

(a) Choose a family (S_r, K_r, J_r) $r \in [0,1]$ that interpolates between the two choices (S_0, K_0, J_0) (S_1, K_1, J_1)

This is possible because the space of choices is contractible.

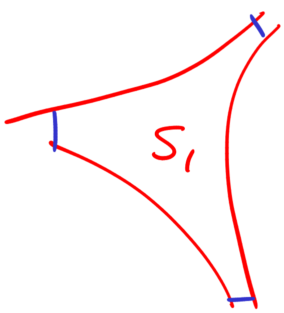
Then $\mathcal{M}_\delta(\{y_s\})^1$ has boundary points of 3 types:



$\rightsquigarrow C\Phi_{S_0}$

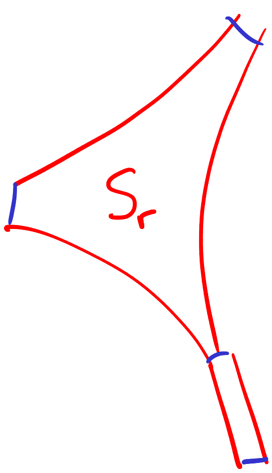
Use $\mathcal{M}_\delta(\{y_s\})^0$ to define an operator P, and then we have

$$C\Phi_{S_1} - C\Phi_{S_0} = \partial P + P\partial$$



$\rightsquigarrow C\Phi_{S_1}$

$C\Phi_{S_1}$ and $C\Phi_{S_0}$ induce same map on cohomology.



$\in \mathcal{M}_\delta(\{y_s\})^0$

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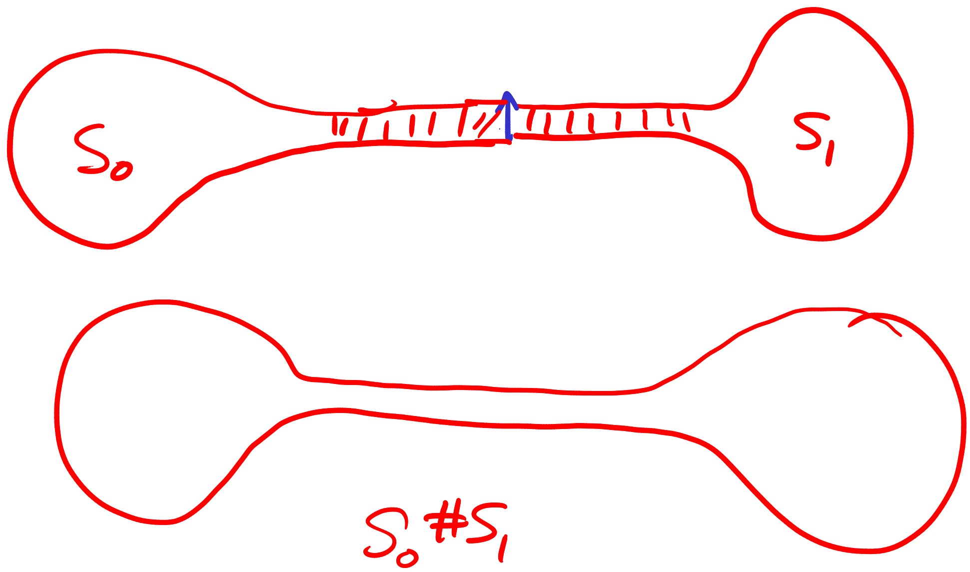
contributes to $P\circ\partial$ or $\partial\circ P$ depending on whether strip is at incoming/outgoing puncture.

(b) Take S_0 and S_1, and glue a strip like end of S_0 to one of S_1



If we choose perturbation data very carefully, we can prove gluing at chain level, but only for this particular choice. By (a), the result holds in cohomology for <u>any</u> choice.

(c) This is now a formal consequence. Take (L_0, L_1) and (H_0, J_0) (H_1, J_1) two choices of Floer data.

Pick a perturbation datum (K_{01}, J_{01}) on $Z = \mathbb{R} \times [0,1]$ interpolating between them, and choose (K_{10}, J_{10}) interpolating in the other direction.

Then $C\Phi_{(Z, K_{01}, J_{01})} : CF^{pr}(L_0, L_1, H_0, J_0) \to CF^{pr}(L_0, L_1, H_1, J_1)$

and similarly for (Z, K_{10}, J_{10})

By gluing $(\Phi_{Z, K_{10}, J_{10}}) \circ (\Phi_{Z, K_{01}, J_{01}}) = Id : HF^{pr}(L_0, L_1, H_0, J_0) \circlearrowleft$

and similarly

$(\Phi_{Z, K_{01}, J_{01}}) \circ (\Phi_{Z, K_{10}, J_{10}}) = Id : HF^{pr}(L_0, L_1, H_1, J_1) \circlearrowleft$

so $HF^{pr}(L_0, L_1, H_0, J_0) \underset{\Phi_{(Z, K_{01}, J_{01})}}{\overset{\Phi_{(Z, K_{10}, J_{10})}}{\rightleftarrows}} HF^{pr}(L_0, L_1, H_1, J_1)$

are isomorphisms.