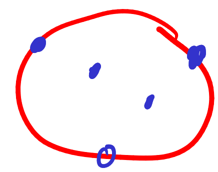
Lecture 12 TFT from symplectic manifolds

The techniques of symplectic geometry and Floer theory provide a geometric source of 2D open/closed TFTs.

Here is an outline that we shall fill in:

- Worldsheets: Riemann surfaces, possibly with boundary and/or punctures



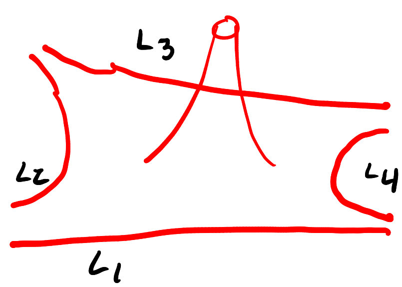
- Target space: Symplectic manifold (X^{2n}, ω)
  $\omega \in \Omega^2(X)$ $\quad d\omega = 0$ $\quad \omega^n > 0$

- D-branes/Boundary conditions: Lagrangian submanifolds
  $L \subset X$, $\dim L = n$, $\omega|_L = 0$.
  (with extra decorations: bundles, ...)

- Closed-string states: $\mathcal{H}_{S^1} = C^*(X)$ cochains on X.

- Open-string states: $\mathcal{H}_{L_1 L_2} = C^*(L_1 \cap L_2)$

- Amplitudes $Z(\Sigma)$



are given by "counting" pseudo-holomorphic maps $u: \Sigma \to X$ with prescribed boundary conditions and asymptotic conditions at the punctures. There is a virtual aspect to this counting process, which at the very least that the curves are counted with signs.