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Holomorphic Anomaly Eqn (Derivation thereof)

Physics $\mathcal{N}=2_{(2,2)}$ SCFT $\rightsquigarrow$ A or B twist.

$$Q = \begin{Bmatrix} Q_A \\ Q_B \end{Bmatrix} \Rightarrow \mathcal{H}_{chiral} \supset \mathcal{H}^{(def)}_{A,B} = \langle \phi_i \rangle_{i=1}^{s}$$

deformation subring

parametrize marginal deformations (deformation is still SCFT)

$Q^2 = 0$

$$A \; (H^{1,1}(X)) = \mathcal{H}^{def}_A$$

$$B \; (H^{2,1}(X)) = \mathcal{H}^{def}_B$$

for (CY_3)

Marginal Def from ϕ_i?

$$S \longrightarrow S + \underset{\underset{\mathbb{C}}{\cap}}{t^i} \int_{\Sigma} d^2 z\, \phi_i^{(2)}$$

$$\phi_i^{(2)} := \{ G^-, [G^-, \phi_i] \}$$

supercurrents

$\mathcal{N}=2$ SCFT $\rightsquigarrow$ Vertex Alg (OPE's)

$$\oint G^-(z) \rightsquigarrow Q^- \qquad \widehat{u(1)}_L \times \widehat{u(1)}_R$$

2

$\cancel{[\mathcal{O}, A(z,\bar{z})]}$

$$[\mathcal{O}, A(z_2,\bar{z}_2)] = \operatorname*{Res}_{z_1\to z_2} j(z_1) A(z_2,\bar{z}_2)$$

holomorphic charge

any op

OPE

$\mathcal{O} = \oint \frac{dz}{2\pi i} j(z)$

$$= \oint_{C_{z_2}} j(z_1) A(z_2,\bar{z}_2)$$

To keep action real, do:

$$S \to S + t^i \int_\Sigma d^2z\, \phi_i^{(2)} + \bar{t}^{\bar{i}} \int_\Sigma d^2z\, \bar{\phi}_{\bar{i}}^{(2)}$$

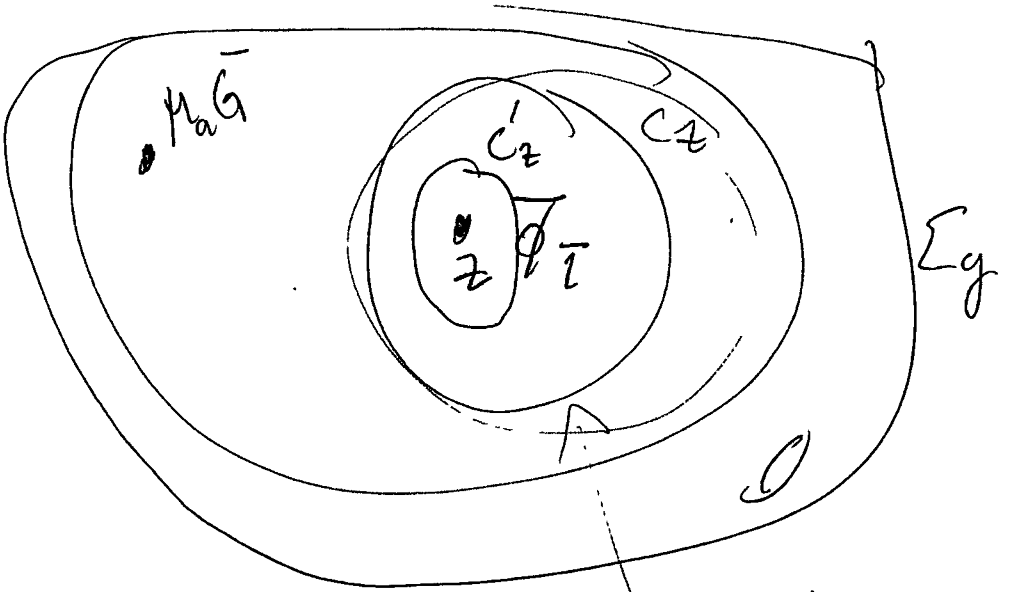
$$\mathcal{F}_g = \int_{\mathcal{M}_g} [dm] \left\langle \prod_{a=1} \int \mu_a G^- \int \bar{\mu}_{\bar{a}} \bar{G}^- \right\rangle_{\Sigma_g}$$

where $\int \mu_a G^- := \int_{\Sigma_g} \mu^{(a)}{}_{\bar{z}}{}^{z}\, G^-_{zz}\, dz \wedge d\bar{z}$

$$\bar{\partial}_{\bar{i}} \mathcal{F}_g = \int_{\mathcal{M}_g} [dm] \int_\Sigma d^2z \left\langle \oint_{C_z} G^+ \oint_{C'_z} \bar{G}^+ \bar{\phi}_{\bar{i}}(z) \prod(\ldots) \right\rangle$$

$$\| $$

$$\frac{\partial}{\partial \bar{t}^{\bar{i}}}$$



can be seen as encircling z,
or as encircling the complement
with opposite orientation.

If $C_z \rightsquigarrow$ surrounding $x^{\mathcal{O}} \Longleftrightarrow \oint_{C_z^*} G^+ \cdot \mathcal{O}$

Do residue calculus.

$G^+(z)\,G^-(w)$ contains $\dfrac{2T(w)}{(z-w)}$ + more singular

$T(w)$ stress energy tensor. (classically $T_{\alpha\beta} \sim \dfrac{\delta S}{\delta \gamma^{\alpha\beta}}$) metric on Σ_g

(modes = Virasoro)

Two copies of $\mathcal{N}=2$ SCA. $(G^+, G^-, T, \dots)$

they commute with each other. $(\bar{G}^+, \bar{G}^-, \bar{T}, \dots)$

$$\oint_{C_z} G^+(w)\,dw \int d^2z\, \mu^{(b)\,z}_{\bar z}\, \bar G_{zz}(z)$$

Commute / use OPE | get $2\int \mu_b T$

$$\int_{\mathcal{M}_g} [dm] \sum_{b=1}^{3g-3} \left\langle \int \bar\phi_{\bar 1} \int 2\mu_b T \int 2\bar\mu_{\bar b} \bar T \prod_{a\neq b} \int \mu_a G^- \int \bar\mu_{\bar a} \bar G^- \right\rangle_{\Sigma_g}$$

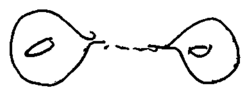
Next trick: insertion of $\int 2\mu_b T \int 2\bar\mu_{\bar b} \bar T \Longleftrightarrow 4 \dfrac{\partial^2}{\partial m_b \partial m_{\bar b}}$

m_b coords on $\mathcal{M}_g$

$$\int_{\mathcal{M}_g} [dm] \sum_{b,\bar b} \frac{4\partial^2}{\partial m_b \partial m_{\bar b}} \left\langle \oint \bar\phi_{\bar 1} \prod \int \mu_a G^- \int \bar\mu_{\bar a} \bar G^- \right\rangle$$

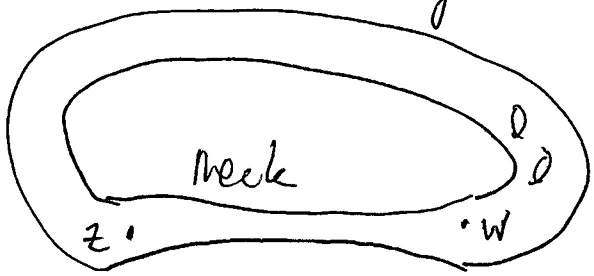
$$\partial \mathcal{M}_g = \coprod_{l=0}^{[g/2]} D_g^{(l)}$$

$D^0_{g=2} \sim$ 

$D^{l\geq 1}_{g=2}$ 

$\Sigma_l \qquad \Sigma_{g-l}$

contrib of D_g^0 close to D_g^0



moduli $m = (m', z, w, \tau)$

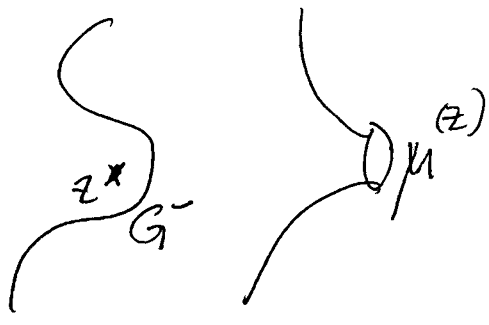
coordinates on $\mathcal{M}_{g-1}$ — punctures — (length, torsion)

$$\int_{\mathcal{M}_g} [dm] \qquad [dm', dw, dz, d\tau]$$

$$\frac{\partial^2}{\partial m_b \, \partial \bar{m}_{\bar{b}}} \text{ contains } \frac{\partial^2}{\partial \tau \, \partial \bar{\tau}} \rightsquigarrow \frac{\partial}{\partial \, \mathrm{Im}(\tau)}$$

$$\int_{\Sigma_g} \mu^{(z)} G^- \rightsquigarrow \oint_{C_z} G^-$$

associated to marked point z

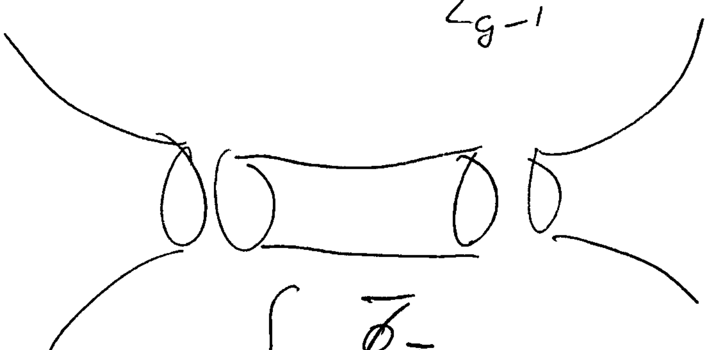


$$\int_{\mathcal{D}^0_g} [dm][dw, dz] \frac{\partial}{\partial\, \mathrm{Im}\,\tau} \Big\langle \int_{\Sigma_g} \bar\phi_{\bar i} \oint_{C_z} G^- \oint_{C_z'} \bar G^- \oint_{C_w} G^- \oint_{C_w'} \bar G^- \prod_{a=1}^{3g-6} \int_{\Sigma_{g-1}} \mu_a' G^- \int_{\Sigma_{g-1}} \bar\mu_a \bar G^- \Big\rangle_{\Sigma_g}$$

This is essentially a residue.

$$\int_{\Sigma_g} \bar\phi_{\bar i} = \int_{\Sigma_{g-1}} \bar\phi_{\bar i} + \int_{tube} \bar\phi_{\bar i}$$

upshot $\int_{\Sigma_{g-1}} = 0$



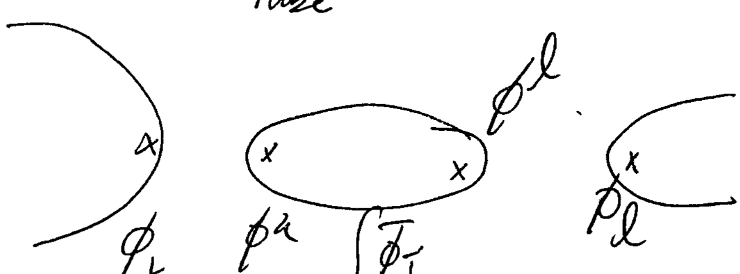
$$\int_{Tube} \bar\phi_{\bar i}$$

metric

$= \langle \phi_i, \phi_j \rangle_{S^2}$

$=: \eta_{ij}$

$$\langle \phi_k, \phi^J \rangle = \delta_k^J$$



only deforming Σ_k are summed over.

$$\to \phi_i(z)\, \eta^{ii'} \langle i' | \int \bar\phi_{\bar i} | k' \rangle \eta^{k'k} \phi_k(w)$$

Get ~~[illegible]~~

Kähler pot + metric

$$\langle j | \bar{\phi}_{\bar{i}} | k \rangle$$

$$= \bar{C}_{\bar{i}\bar{j}\bar{k}} \, e^{2K} G^{\bar{j}j'} G^{\bar{k}k'} \eta_{jj'} \eta_{kk'}$$

independent of where $\bar{\phi}_{\bar{i}}$ is, so

$\{G^-[G^-, \bar{\phi}]\}$

$$\bar{C}_{\bar{i}\bar{j}\bar{k}} \, e^{2K} G^{j\bar{j}} G^{k\bar{k}} \int_{M_{g-1}} [dm'] \Big\langle \int_{\Sigma_{g-1}} \phi_j^{(2)} \int_{\Sigma_{g-1}} \phi_k^{(2)} \prod_{a=1}^{3g-6} \int \mu'_a G^- \int \bar{\mu}'_a \bar{G}^- \Big\rangle_{\Sigma_{g-1}}$$

$$\Big\langle \int \phi_j^{(2)} \int \phi_k^{(2)} \prod_{a=1}^{3g-6} \int \mu'_a G^- \int \bar{\mu}'_a \bar{G}^- \Big\rangle_{\Sigma_{g-1}}$$

$$\parallel$$

$$D_j D_k F_{g-1}$$

cov. of ∂.