Maurizio Romo BCOV eqns and higher genus MS. 1

Set up/Intro. Higher genus Part. Func.

$\rightsquigarrow$ coupling w/ Topological gravity.

$$\mathcal{F}_g = \int_{\mathcal{M}_g} \left\langle \left| \prod_{i=1}^{3g-3} G^-(\mu^{(i)}) \right|^2 \right\rangle$$

G^- is a field.

moduli of genus g Riemann surfaces.

$$G^-(\mu^{(a)}) = \left(\int_{\Sigma_g} G^- \cdot \underbrace{\mu^{(a)}}_{} \right) dm^{(a)} \in T^*\mathcal{M}_g$$

Beltrami Diff'l $H^{(0,1)}(T_{\Sigma_g})$

2-form valued operators.

$$G^-_{\mu\nu} (\mu^{(a)})^{\nu}_{\bar{\delta}}\, dx^{\mu} \wedge dx^{\bar{\delta}}$$

quadratic diff

$H^0(K^2)$

$\parallel$

$H^1(K \otimes K^{-2})^{\vee} = H^1(K^{-1})$

$$\mathcal{F} := \sum \lambda^{2-2g} \mathcal{F}_g \qquad \text{Free energy}$$

Kodaira-Spencer theory of gravity

M CY 3-fold

(B-model) $\quad A \in H^{(0,1)}(T_M) \qquad \bar\partial A + \frac{1}{2}[A,A] = 0$

field (on M) $\qquad A \to A + \bar\partial \varepsilon$

$$A' = \Omega \cdot A \in H^{(3,1)}(M) \qquad \partial A' = 0$$

$\bar\partial^{\dagger} A' = 0$ (gauge fixing)

(BCOV) given harmonic form $x \in H^{(0,1)}(T_M) \xrightarrow{\text{unique}} A[x]$

$$S(A,x \mid \underbrace{t,\bar t}_{\text{complex str on } M}) = \frac{1}{\lambda^2}\left[\frac{1}{2}\int A' \frac{1}{\partial}\bar\partial A' + \frac{1}{6}\int_M \big((x+A)\wedge(x+A)\big)' \wedge (x+A)'\right]$$

$$\int DA \, e^{S(A,x \mid t,\bar t)}$$

$$C^{(g)}_{i_1 \cdots i_m} := D_{i_1} \cdots D_{i_m} \mathcal{F}_g(t, \bar{t})$$

covariant derivative on $\mathcal{M}_g$

$$D_i = \frac{D}{Dt_i}$$

$$W(\lambda, x \mid t, \bar{t}) = \sum_{g=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{n!} \lambda^{2g-2} C^{(g)}_{i_1 \cdots i_n} x^{i_1} \cdots x^{i_n}$$

$$+ \left(\cancel{\tfrac{\chi(M)}{2}} \frac{\chi(M)}{24} - 1 \right) \ln(\lambda)$$

$$\int DA \, e^{S(A, x \mid t, \bar{t})} = e^{W(\lambda, x \mid t, \bar{t})}$$

Gauge $\Longleftrightarrow$ string. duality.

$$\bar{\partial}_{\bar{\ell}} \mathcal{F}_g = \cdots R\left(\mathcal{F}_{h<g}, D\mathcal{F}_{h<g}, DD\mathcal{F}_{h<g}\right)$$

hol anomaly: valid for any $N=(2,2)$ SCFT

or $N=(2,2)$ w/ $U(1)_R$-symmetry

e.g. in A-model.

Where these $\mathcal{F}_g$ lives?

$t, \bar{t} \in \mathcal{M}_{cs.}$ (or $\mathcal{M}_{\text{Kähler}}$ in A-model)

$\mathcal{M}$ special Kähler geometry.

<u>Def</u>: A Hodge manifold M is a compact Kähler manifold w/ hermitian line bundle $\mathcal{L} \to M$ s.t. Kähler potential for Kähler metric is

$$K = -\log \langle \Omega, \bar{\Omega} \rangle$$

hol section of $\mathcal{L}$.

<u>Def</u>: Special Kähler M is a Hodge manifold

s.t. 1) $\exists$ V.B. $\mathcal{H} = \mathcal{L} \oplus (\mathcal{L} \otimes \underset{T^{(1,0)}M}{\underset{\parallel}{TM}}) \oplus \overline{\mathcal{L} \otimes TM} \oplus \bar{\mathcal{L}}$

w/ connection. $\nabla : \Gamma(M, \mathcal{H}) \to \Gamma(M, \mathcal{H}) \otimes \Omega^1_M$

s.t. ∇ is flat and

$$\nabla_i \begin{pmatrix} \xi_0 \\ \xi_j \\ \xi_{\bar{j}} \\ \xi_{\bar{0}} \end{pmatrix} = (\partial_i + A)\, \xi$$

upper triangular

$$\mathcal{F}_g(t,\bar{t}) \in \Gamma_{C^\infty}(\mathcal{M}, \mathcal{L}^{2-2g})$$

E.g. when $\mathcal{M} = \mathcal{M}_{cs}(M)$ for M - CY3

$$\mathcal{L} = H^{(3,0)}(M_t, \mathbb{C})$$

$$\mathcal{H} := H^3(M_t, \mathbb{C})$$

metric = Weil-Petersson metric.

$$K = -\log\left(\int_M \Omega_M(t) \wedge \overline{\Omega}_M(t)\right)$$

∇ = Gauss-Manin connection

$\mathcal{M}$ = exactly marginal deformations.

$\mathcal{L}$ = vacuum bundle.

<u>Hol anomaly</u>: $g \geq 2$: $\bar{\partial}_{\bar{i}} \mathcal{F}_g = \frac{1}{2} \overline{C}_{\bar{i}}^{jk} \Big(D_j D_k \mathcal{F}_{g-1}$

$$+ \sum_{r=1}^{g-1} D_j \mathcal{F}_r D_k \mathcal{F}_{g-r} \Big)$$

$\overline{C}_{\bar{i}}^{jk}$ = Yukawa couplings.

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$$\partial_i \bar{\partial}_{\bar{j}} \mathcal{F}_1 = \frac{1}{2} C_{ijk} \bar{C}_{\bar{j}}^{\bar{k}\bar{l}} + \left(1 + \frac{\chi}{24}\right) G_{i\bar{j}}$$

$\uparrow$

1 primitive

Bohan: Givental quantization
graph sum

Minimal again

Thurs/Fri okay.

Thus for Bohan

~~[illegible]~~

Thus symplectic sum