MATH 285 HOMEWORK 5 SOLUTIONS

Section 3.2

22. Imposing the initial conditions $y(0) = 0$ and $y'(0) = 10$ in the general solution $y(x) = c_1e^{2x}+c_2e^{-2x}-3$ yields the equations $c_1+c_2-3 = 0$, $2c_1 - 2c_2 = 10$. The solution is $c_1 = 4$, $c_2 = -1$. The desired particular solution is $y(x) = 4e^{2x} - e^{-2x} - 3$.
23. Imposing the initial conditions $y(0) = 3$ and $y'(0) = 11$ on the general solution $y(x) = c_1e^{-x} + c_2e^{3x} - 2$ yields the equations $c_1 + c_2 - 2 = 3$ and $-c_1 + 3c_2 = 11$. The solution is $c_1 = 1$, $c_2 = 4$. The desired particular solution is $y(x) = e^{-x} + 4e^{3x} - 2$.
24. The general solution is $y(x) = c_1e^x \cos x + c_2e^x \sin x + x + 1$, and its derivative is $y'(x) = c_1(e^x \cos x - e^x \sin x) + c_2(e^x \sin x + e^x \cos x) + 1$. Imposing the initial conditions $y(0) = 4$ and $y'(0) = 8$ on the general solution yields the equations $c_1 + 1 = 4$ and $c_1 + c_2 + 1 = 8$. The solution is $c_1 = 3$, $c_2 = 4$. The desired particular solution is $y(x) = 3e^x \cos x + 4e^x \sin x + x + 1$.

Section 3.3

3. The characteristic equation is $r^2 + 3r - 10 = 0$. This factors as $(r + 5)(r - 2) = 0$. The roots are $r = -5, 2$, which are real and distinct, so the general solution of the differential equation is $y(x) = c_1e^{-5x} + c_2e^{2x}$.
4. The characteristic equation is $2r^2 - 7r + 3 = 0$. The quadratic formula gives $r = (7 \pm \sqrt{49 - 24})/4 = (7 \pm 5)/4 = 1/2, 3$. These are real and distinct, so the general solution of the differential equation is $y(x) = c_1e^{x/2} + c_2e^{3x}$.
6. The characteristic equation is $r^2+5r+5 = 0$. The quadratic formula gives $r = (-5 \pm \sqrt{25 - 20})/2 = (-5 \pm \sqrt{5})/2$. The roots are real and distinct, so the general solution of the differential equation is $y(x) = c_1e^{(-5+\sqrt{5})x/2} + c_2e^{(-5-\sqrt{5})x/2}$.
7. The characteristic equation is $4r^2 - 12r + 9 = 0$. The quadratic formula gives $r = (12 \pm \sqrt{12^2 - 4 \cdot 4 \cdot 9})/8 = 12/8 = 3/2$. Thus there is a single root of multiplicity two. The general solution is therefore $y(x) = c_1e^{3x/2} + c_2xe^{3x/2}$.
11. The characteristic equation is $r^4 - 8r^3 + 16r^2 = 0$. Factoring out r^2 gives $r^2(r^2 - 8r + 16) = 0$, and $r^2 - 8r + 16 = (r - 4)^2$. Thus the characteristic equation is $r^2(r - 4)^2 = 0$, which has two roots $r = 0$ and $r = 4$, each with multiplicity two. Therefore the general solution is $y(x) = c_1 + c_2x + c_3e^{4x} + c_4xe^{4x}$.

15. The characteristic equation is $r^4 - 8r^2 + 16 = 0$. Writing this as $(r^2)^2 - 8(r^2) + 16 = 0$, we can think of it as a quadratic equation for r^2. The solution is then $r^2 = 4$, and $r = 2, -2$, each of which is a repeated root. The full factorization is $r^4 - 8r^2 + 16 = (r^2 - 4)^2 = (r-2)^2(r+2)^2$. The general solution is $y(x) = c_1 e^{2x} + c_2 x e^{2x} + c_3 e^{-2x} + c_4 x e^{-2x}$.
39. In order for $y(x) = (A + Bx + Cx^2)e^{2x}$ to be the general solution, the characteristic equation would need to have a root $r = 2$ of multiplicity three. For instance, $(r-2)^3 = 0$. Expanding this out gives $r^3 - 6r^2 + 12r - 8 = 0$. The corresponding differential equation is $y''' - 6y'' + 12y' - 8y = 0$.