# MATH 285 HOMEWORK 4 SOLUTIONS

## Section 3.1

3. We have that $y_1' = -2\sin 2x$ and $y_1'' = -4\cos 2x$. Thus $y_1'' + 4y_1 = -4\cos 2x + 4\cos 2x = 0$. Similarly $y_2'' = -4\sin 2x$, so $y_2'' + 4y_2 = -4\sin 2x + 4\sin 2x = 0$. Imposing the initial conditions $y(0) = 3$ and $y'(0) = 8$ on the general solution $y(x) = c_1\cos 2x + c_2\sin 2x$ yields the equations

$$3 = y(0) = c_1, \quad 8 = y'(0) = 2c_2$$

with solution $c_1 = 3$, $c_2 = 4$. Hence the particular solution is $y(x) = 3\cos 2x + 4\sin 2x$.

13. We have that $y_1' = 1$ and $y_1'' = 0$, so $x^2y_1'' - 2xy_1' + 2y_1 = 0 - 2x + 2x = 0$. Also, $y_2' = 2x$ and $y_2'' = 2$, so $x^2y_2'' - 2xy_2' + 2y_2 = 2x^2 - 2x(2x) + 2x^2 = 0$. Thus the general solution is $y(x) = c_1x + c_2x^2$. Imposing the initial conditions $y(1) = 3$ and $y'(1) = 1$ yields the equations

$$3 = y(1) = c_1 + c_2, \quad 1 = y'(1) = c_1 + 2c_2$$

Solving this system gives $c_1 = 5$, $c_2 = -2$. Hence the particular solution is $y(x) = 5x - 2x^2$.

24. A trig identity for $\cos 2x$ says

$$\cos 2x = 1 - 2\sin^2 x$$

meaning that $2\sin^2 x = 1 - \cos 2x$. So if $f(x) = \sin^2 x$ and $g(x) = 1 - \cos 2x$, then $g(x) = 2f(x)$. Since $f(x)$ and $g(x)$ are proportional, they are linearly dependent.

33. The characteristic equation is $r^2 - 3r + 2 = 0$. The solutions are $r = 1, 2$. Thus $y_1(x) = e^x$ and $y_2(x) = e^{2x}$ are solutions, and the general solution is $y(x) = c_1e^x + c_2e^{2x}$.

35. The characteristic equation is $r^2 + 5r = 0$. The solutions are $r = 0, -5$. Thus $y_1(x) = e^{0x} = 1$ and $y_2(x) = e^{-5x}$ are solutions, and the general solution is $y(x) = c_1 + c_2e^{-5x}$.

46. We work backwards from the solution $y(x) = c_1e^{10x} + c_2e^{100x}$. The roots of the characteristic equation must be $r = 10, 100$, and such a characteristic equation would be $(r-10)(r-100) = 0$, or $r^2 - 110r + 1000 = 0$. Thus $y'' - 110y' + 1000y = 0$ is a differential equation with the original $y(x)$ as its general solution.

## Section 3.2

3. Take $1 \cdot f(x) + 0 \cdot g(x) + 0 \cdot h(x) = 1 \cdot 0 + 0 \cdot \sin x + 0 \cdot e^x = 0$.
5. We know that $\cos 2x = 2\cos^2 x - 1$. Multiply this equation by 17 to get $17 \cos 2x = 34 \cos^2 x - 17$. Thus $17 - 34\cos^2 x + 17\cos 2x = 0$. Thus we find
$$1 \cdot f(x) + (-34) \cdot g(x) + 17 \cdot h(x) = 0$$
9. Suppose that $c_1, c_2, c_3$ are constants such that $c_1 e^x + c_2 \cos x + c_3 \sin x = 0$ for all values of $x$. Plugging in $x = 0$ gives $c_1 + c_2 = 0$. Taking the derivative gives $c_1 e^x - c_2 \sin x + c_3 \cos x = 0$, and plugging in $x = 0$ then gives $c_1 + c_3 = 0$. Taking the derivative again gives $c_1 e^x - c_2 \cos x - c_3 \sin x = 0$, and plugging in $x = 0$ gives $c_1 - c_2 = 0$. So we have the three equations
$$c_1 + c_2 = 0, \quad c_1 + c_3 = 0, \quad c_1 - c_2 = 0.$$
The third implies $c_1 = c_2$. Using this in the first we get $c_1 = 0$ and $c_2 = 0$. Then the second equation implies $c_3 = 0$ as well. Since the three constants $c_1, c_2, c_3$ are forced to be zero, $e^x, \cos x$, and $\sin x$ are linearly independent.
13. The general solution is $y(x) = c_1 e^x + c_2 e^{-x} + c_3 e^{-2x}$. The derivatives are $y'(x) = c_1 e^x - c_2 e^{-x} - 2c_3 e^{-2x}$ and $y''(x) = c_1 e^x + c_2 e^{-x} + 4c_3 e^{-2x}$. The initial conditions $y(0) = 1$, $y'(0) = 2$, $y''(0) = 0$ imply
$$1 = c_1 + c_2 + c_3, \quad 2 = c_1 - c_2 - 2c_3, \quad 0 = c_1 + c_2 + 4c_3$$
Subtracting the first equation from the third gives $-1 = 3c_3$, so $c_3 = -1/3$. Then the equations become
$$c_1 + c_2 = 4/3, \quad c_1 - c_2 = 4/3$$
Thus $c_1 = 4/3$ and $c_2 = 0$. The desired particular solution is $y(x) = (4/3)e^x - (1/3)e^{-2x}$.