MATH 285 HOMEWORK 1 SOLUTIONS

Section 1.1

3. If $y_1 = \cos 2x$ and $y_2 = \sin 2x$, then $y_1' = -2\sin 2x$ and $y_2' = 2\cos 2x$. Then $y_1'' = -4\cos 2x = -4y_1$ and $y_2'' = -4\sin 2x = -4y_2$. Thus $y_1'' + 4y_1 = 0$ and $y_2'' + 4y_2 = 0$.
7. If $y_1 = e^x \cos x$ and $y_2 = e^x \sin x$, then the first derivatives are $y_1' = e^x \cos x - e^x \sin x$ and $y_2' = e^x \sin x + e^x \cos x$. The second derivatives are

$$y_1'' = e^x \cos x - e^x \sin x - e^x \sin x - e^x \cos x = -2e^x \sin x$$
$$y_2'' = e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x = 2e^x \cos x$$

Thus

$$y_1'' - 2y_1' + 2y_1 = -2e^x \sin x - 2(e^x \cos x - e^x \sin x) + 2e^x \cos x = 0$$
$$y_2'' - 2y_2' + 2y_2 = 2e^x \cos x - 2(e^x \sin x + e^x \cos x) + 2e^x \sin x = 0$$

19. If $y(x) = Ce^x - 1$, then $y' = Ce^x = y + 1$. If $y(x) = Ce^x - 1$ and $y(0) = 5$, we have $5 = Ce^0 - 1 = C - 1$, so $C = 6$.
35. $\frac{dN}{dt} = k(P - N)$
43. (a) If $x(t) = 1/(C - kt)$, then $\frac{dx}{dt} = (-k)(-1/(C-kt)^2) = k/(C - kt)^2 = kx^2$. (b) If $x(t) = 0$, then $\frac{dx}{dt} = 0$, so indeed $\frac{dx}{dt} = kx^2$. So $x(t) = 0$ solves the initial value problem $x(0) = 0$.

Section 1.2

17. If $a(t) = (t+1)^{-3}$, then $v(t) = \int (t+1)^{-3} dt = -(1/2)(t+1)^{-2} + C$. In order to have $v(0) = 0$, we take $C = 1/2$. Then

$$x(t) = \int [-(1/2)(t+1)^{-2} + (1/2)]dt = (1/2)(t+1)^{-1} + (1/2)t + C$$

In order to have $x(0) = 0$, we take $C = -1/2$. So

$$x(t) = \frac{1}{2}[(t+1)^{-1} + t - 1]$$

20. The graph of $v(t)$ shows that

$$v(t) = \begin{cases} t & \text{if } 0 \le t \le 5 \\ 5 & \text{if } 5 \le t \le 10 \end{cases}$$

This means that

$$x(t) = \begin{cases} t^2/2 + C_1 & \text{if } 0 \le t \le 5 \\ 5t + C_2 & \text{if } 5 \le t \le 10 \end{cases}$$

In order to have $x(0) = 0$, we must have $C_1 = 0$. In order for $x(t)$ to be continuous, $x(t) = t^2/2$ and $x(t) = 5t + C_2$ must agree when $t = 5$. This implies that $C_2 = -25/2$.

27. In units of meters and seconds, $a = -9.8$, so $v = -9.8t - 10$ and $y = -4.9t^2 - 10t + y_0$, where y_0 is the height of the building. When the ball hits the ground, $y = 0$ and $v = -60$. Solving $-60 = -9.8t - 10$, we find that $t \approx 5.10$ is the time it takes to hit the ground. Using the equation for position,

$$0 = -4.9(5.10)^2 - 10(5.10) + y_0$$

So $y_0 \approx 178.57$.

33. Let a denote the acceleration due to gravity on Gzyx, which is unknown. When a ball is dropped from a height of 20 feet, we have $v = -at$ and $y = -at^2/2 + 20$. We know the ball takes 2 seconds to hit the ground at $y = 0$, so

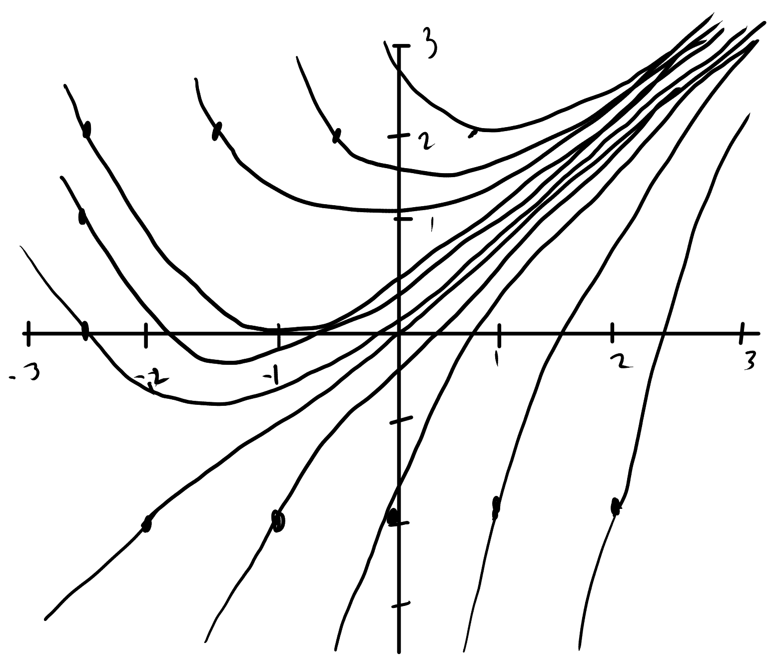
$$0 = -a(2)^2/2 + 20$$

and we find $a = 10$.

Dropping a ball from 200 feet, we have $v = -10t$ and $y = -5t^2 + 200$. It hits the ground when $0 = -5t^2 + 200$, or $t = \sqrt{40} = 2\sqrt{10}$. The velocity is then $v = -20\sqrt{10}$.

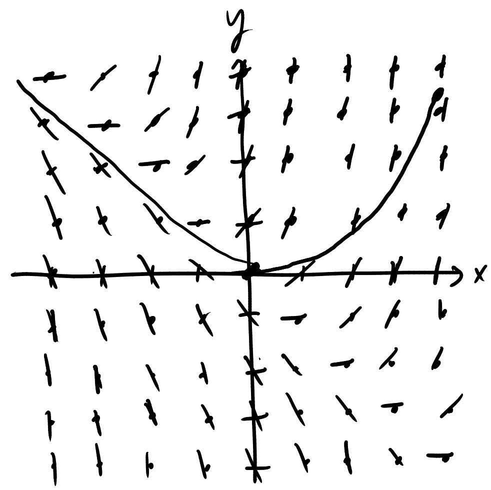
Section 1.3

6. Since this is only approximate, answers may vary.



21. Answers may vary. $y(-4) \approx 3$.

y \ x	-4	-3	-2	-1	0	1	2	3	4
4	0	1	2	3	4	5	6	7	8
3	-1	0	1	2	3	4	5	6	7
2	-2	-1	0	1	2	3	4	5	6
1	-3	-2	-1	0	1	2	3	4	5
0	-4	-3	-2	-1	0	1	2	3	4
-1	-5	-4	-3	-2	-1	0	1	2	3
-2	-6	-5	-4	-3	-2	-1	0	1	2
-3	-7	-6	-5	-4	-3	-2	-1	0	1
-4	-8	-7	-6	-5	-4	-3	-2	-1	0



24. If $y(x) = kx$, then $y' = k$, and indeed $xy' = kx = y$.. The initial value problem $y(a) = b$ has one solution if $a \neq 0$, and if $a = 0$, then it has either infinitely many solutions (if $b = 0$ also) or no solutions (if $b \neq 0$).