NAME:

NetID:

MATH 285 G1 Exam 3 (B) **April 18, 2016** **Instructor: Pascaleff**

Problem	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

INSTRUCTIONS:

- Do all work on these sheets.
- Show all work.
- No books, notes, or calculators.

Orthogonality formulas

$$\int_{-L}^{L} \cos \frac{m\pi t}{L} \cos \frac{n\pi t}{L} \, dt = \begin{cases} 0, & m \neq n \\ L, & m = n \end{cases} \tag{1}$$

$$\int_{-L}^{L} \sin \frac{m\pi t}{L} \sin \frac{n\pi t}{L} \, dt = \begin{cases} 0, & m \neq n \\ L, & m = n \end{cases} \tag{2}$$

$$\int_{-L}^{L} \cos \frac{m\pi t}{L} \sin \frac{n\pi t}{L} \, dt = 0 \tag{3}$$

Some integral formulas

$$\int u \cos u \, du = u \sin u + \cos u + C \tag{4}$$

$$\int u \sin u \, du = -u \cos u + \sin u + C \tag{5}$$

1. (20 points) An undamped oscillator with mass $m = 3$ and spring constant $k = 9$ is driven by a driving force $F(t)$ which is given as a Fourier series

$$F(t) = \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n\pi t + \frac{2}{n} \sin n\pi t$$

The differential equation for $x(t)$ is

$$3x'' + 9x = F(t)$$

Find a particular solution of this equation.

2. (a) (10 points) Suppose that a function $f(t)$ which is periodic of period 2π has the Fourier series

$$f(t) = \sum_{n=1}^{\infty} \frac{1}{4n + n^2} \cos nt$$

Use the orthogonality formulas to evaluate the integral

$$\int_{-\pi}^{\pi} f(t) \cos 3t \, dt$$

(b) (10 points) Let $g(t)$ be the function which is periodic of period 16, and which is defined on the interval $-8 \leq t < 8$ by the formula

$$g(t) = 3t + 2t^2 + 4\cos(t)$$

Set up, but do not evaluate, an integral expression for the coefficient of $\cos \frac{3\pi t}{8}$ in the Fourier series of $g(t)$ (also known as a_3 in our standard notation).

3. (a) (5 points) Consider the function which is periodic of period 2π defined on the interval $-\pi \le t < \pi$

$$f(t) = \begin{cases} 22, & -\pi \le t < 0 \\ 2875/\pi^2, & t = 0 \\ 10, & 0 < t < \pi \end{cases}$$

If we take the Fourier series of $f(t)$, and put $t = 0$ in that series, what number does it converge to? Put another way, what is the sum of the Fourier series of $f(t)$ at $t = 0$? Explain your answer (briefly).

(b) (15 points) Consider the function defined by the Fourier series

$$g(t) = \sum_{n=1}^{\infty} 4e^{-2\pi n} \cos n\pi t$$

Find a Fourier series expression for the antiderivative $\int g(t)\, dt$. You are *not* expected to address the question of convergence.

4. (20 points) Let $f(t)$ be an even periodic function of period 4 such that, on the interval $0 < t < 2$,

$$f(t) = -4t, \quad 0 \le t < 2$$

Find the Fourier series of $f(t)$.

5. (20 points) Consider the eigenvalue problem

$$\begin{cases} y'' + \lambda y = 0 \\ y(0) = 0 \\ y'(10) = 0 \end{cases}$$

Find the eigenvalues, and find a single nonzero eigenfunction associated to each eigenvalue. You may assume that all the eigenvalues are positive, for indeed they are.

This page is for work that doesn't fit on other pages.