NAME:

NetID:

MATH 285 G1 Exam 2 (B)   **March 18, 2016**   **Instructor: Pascaleff**

Problem	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

INSTRUCTIONS:

- Do all work on these sheets.

- Show all work.

- No notes, books, calculators, or other electronic devices are permitted.

1. (20 points) Show that the functions $f(x) = 2$ and $g(x) = \cos^2 x$ and $h(x) = 3\sin^2 x$ are *not* linearly independent. That is, find constants A, B, C, *not all zero*, such that

$$Af(x) + Bg(x) + Ch(x) = 0 \text{ for all } x$$

2. (20 points) For each constant coefficient differential operator $p(D)$, find the solutions to the homogeneous differential equation $p(D)y = 0$, where $D = \frac{d}{dx}$. It is not necessary to rederive the solution completely, but keep in mind that partial credit can be given if substantial work is shown. In each part, you are asked to find the general real-valued (not complex-valued) solution.

(a) $p(D) = D^2 - 4D + 3$. Find the general real solution of $p(D)y = 0$.

(b) $p(D) = (D - 3)^2(D + 2)$. Find the general real solution of $p(D)y = 0$.

(c) $p(D) = D^2 + 3D + 5$. Find the general real solution of $p(D)y = 0$.

(d) $p(D) = (D-5)^2(D^2+4)^2$. Find the general real solution of $p(D)y = 0$.

3. (20 points) Consider a mass-spring system with forcing, where the forcing function is a sine with angular frequency ω: the position $x(t)$ obeys the differential equation

$$m\frac{d^2x}{dt^2} + kx = \sin\omega t$$

where the mass m and spring constant k are given by $m = 4, k = 12$.

(a) (15 points) Find a solution of the form $x(t) = A\sin\omega t$ for those values of ω for which it exists.

(b) (5 points) For which values of the driving frequency ω does this system *have no solution* of the form $x(t) = A\sin\omega t$?

4. (20 points) Find the *general solution* of the differential equation

$$y'' + 5y' + 6y = 3x + 1$$

5. (20 points)

   (a) (10 points) Find an annihilator $A(D)$ for $f(x) = 3xe^{2x}$. (That is, find a constant coefficient differential operator $A(D)$ such that $A(D)[f(x)] = 0$.)

   (b) (10 points) Use the annihilator method to find a particular solution of the differential equation $y' - 2y = 3xe^{2x}$.