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MATH 285 G1 Exam 1 (C)

February 17, 2016

Instructor: Pascaleff

<p><b>INSTRUCTIONS:</b></p> <ul style="list-style-type: none"><li>• Do all work on these sheets.</li><li>• Show all work.</li><li>• No notes, books, calculators, or other electronic devices are permitted.</li></ul>
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Problem	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20 points) Consider the differential equation

$$\frac{dy}{dx} = (x + 3)y$$

- (a) (10 points) Draw a slope field for this equation.

- (b) (10 points) Given the initial condition  $y(1) = 1$ , use Euler's method with two steps to approximate  $y(1.2)$ .

2. (20 points) Let  $y(x)$  be a solution of the initial value problem

$$\frac{dy}{dx} = 1 + y^2, \quad y(0) = 1$$

Starting from  $y_0(x) = 1$ , compute the first and second Picard approximations  $y_1(x)$  and  $y_2(x)$ , and use  $y_2(x)$  to estimate  $y(0.1)$ .

3. (20 points) Find the general solution of

$$\frac{dy}{dx} = e^{5x} - 2y$$

4. (20 points) Let  $P(t)$  be denote a population of fish in a lake. This population is governed by the differential equation

$$\frac{dP}{dt} = P(300 - P) - 500$$

- (a) (10 points) Find the equilibrium solutions, and determine whether each is stable or unstable.

- (b) (10 points) Draw a qualitative plot of the solutions of this differential equation.

5. (20 points) A metal ball has been heated to  $400^{\circ}\text{C}$ . It is placed into a bath of water at  $30^{\circ}\text{C}$ . After 5 seconds, it has cooled to a temperature of  $200^{\circ}\text{C}$ .

Suppose now that the metal ball is cooled to  $0^{\circ}\text{C}$ , and again placed into a bath of water at  $30^{\circ}\text{C}$ . How long will it take to reach a temperature of  $20^{\circ}\text{C}$ ? Your answer does not need to be simplified.

In both situations, the process is governed by Newton's law of cooling:

$$\frac{dT}{dt} = -k(T - A)$$

where  $A$  is the temperature of the water, and  $k$  is a constant.

This page is for work that doesn't fit on the other pages.