DIFFERENTIABLE MANIFOLDS II: HOMEWORK 10

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(1) Let $n = 3$ and consider submanifolds $V(f)$ in $\mathbb{CP}^3$ defined by the equation $f(x_0, x_1, x_2, x_3) = 0$, where f is a homogeneous polynomial of degree d. Assuming it is smooth, the manifold $V(f)$ has complex dimension two and is therefore known as a complex surface. Find the Chern classes and Euler characteristic in the cases $d = 2, 3, 4$ (known as quadric, cubic, and quartic surfaces respectively).

(2) Again let $n = 3$ and consider submanifolds $V(f_1, f_2)$ in $\mathbb{CP}^3$ defined *two* equations $f_1 = f_2 = 0$, where f_1 and f_2 are homogeneous polynomials of degrees d_1 and d_2. Since $V(f_1, f_2)$ has complex dimension one it is called complex curve (also a Riemann surface). Find the chern classes of $V(f_1, f_2)$ as a function of d_1 and d_2. Using the fact that the integral of $h \in H^2(\mathbb{CP}^3)$ over $V(f_1, f_2)$ is $d_1 \cdot d_2$, find the Euler characteristic and genus of $V(f_1, f_2)$.