Applications, including Arnold's conjecture for aspherical M.

We have seen that $HF(L,L) = H^*(L)$ for $L \subset T^*L$

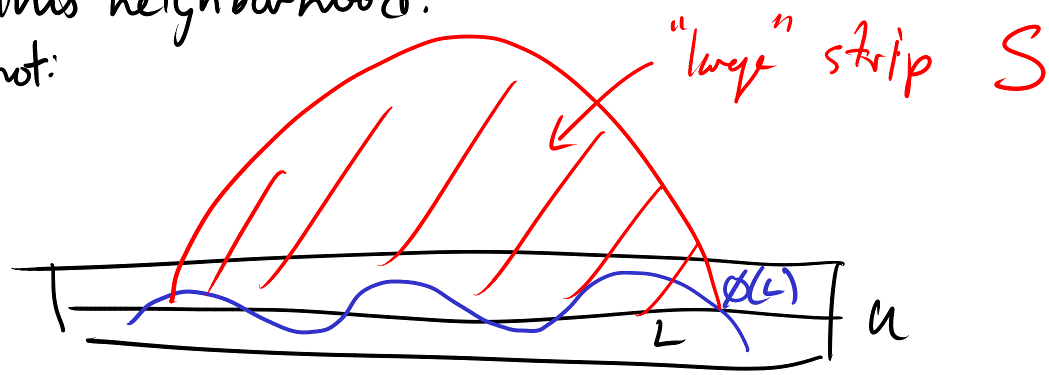
The same conclusion holds under the hypotheses of asphericity $\langle \omega, \pi_2(M,L) \rangle = 0$, as shown by Floer.

The idea is that locally near L, M looks like T^*L by the Weinstein tubular neighborhood theorem. Some contribution to the Differential comes from strips contained in this neighborhood. If there are other contributions, there must be "large" strips, but these are ruled out by asphericity.

Consider the pair $L, \phi(L)$, where $\phi(L)$ is a small transverse hamiltonian push-off.

L and $\phi(L)$ are contained in a Weinstein tubular nbhd $\mathcal{U}$
$L, \phi(L) \subset \mathcal{U} \subset M$ $\qquad$ $\mathcal{U} \hookrightarrow T^*L$ symplectically.

We must prove all strips contributing to the differential stay in this neighborhood.
Suppose not:



The symplectic area of S is positive, becaus it is J-holomorphic.

$$\int_S \omega > 0.$$

Take smaller and smaller pushoffs ϕ
If the large strip persists, by gromov compactness we will obtain a disk with boundary on L in the limit.
contradicts $\langle \omega, \pi_2(M,L) \rangle = 0$.

Thus for small enough ϕ, the computation goes through as before.

NB: The issue of hamiltonian invariance in the nonexact setting (even aspherical) is somewhat delicate.

It is possible to show that $HF(L, \phi(L))$ does not depend on the Hamiltonian diffeo ϕ of $\langle \omega, \pi_2(M,L) \rangle = 0$
But some thing like $HF(L_0, \phi(L_1))$ could depend on ϕ even if everything is aspherical, unless we work over an appropriate Novikov field.

e.g.

Arnold's Conjecture (special case)

<u>Thm</u> Assume $\langle \omega, \pi_2(M)\rangle = 0$. Let $\phi \in Ham(M,\omega)$ be a diffeo generated by a time-dependent hamiltonian. Assume all fixed points of ϕ are non-degenerate. Then

$$\# Fix(\phi) \geq \sum \text{rank } H_i(M;\mathbb{Z}_2)$$

<u>Rk</u> Lefschetz yields $\# Fix \geq \chi(M)$, so this is better if $H_{odd}(M;\mathbb{Z}_2) \neq 0$.

<u>Proof</u> Consider $(M \times M, (-\omega)\times(\omega))$

$\Delta = \{(x,x) \mid x \in M\} \subset M\times M$ is Lagrangian.

So is $graph(\phi) = \{(x,\phi(x)) \mid x \in M\}$

Also, graph ϕ is hamiltonian isotopic to Δ, via the same hamiltonian placed on the second M-factor.

Nondegeneracy means $\Delta \pitchfork graph(\phi)$

Claim $\langle \omega, \pi_2(M\times M, \Delta)\rangle = 0$ (doubling trick)

Thus $HF(\Delta, graph(\phi)) = HF(\Delta,\Delta) = H_*(\Delta) = H_*(M)$ ▨

Another application: Let $L \overset{\text{compact}}{\hookrightarrow} \mathbb{C}^n$ be a Lagrangian embedding. Then $\langle \omega, \pi_2(M,L) \rangle \neq 0$
in particular, L cannot be exact. (Gromov '85)

This result is interesting because it contrasts with the Whitney embedding theorem and the h-principle for exact Lagrangian immersions.

Whitney: any n-manifold
- immerses in $\mathbb{R}^{2n}$ (easy)
- embeds in $\mathbb{R}^{2n}$ (harder)

Gromov: Let $(W, \omega = d\theta)$ be an exact symplectic manifold. Then the h-principle holds for exact Lagrangian immersions $V \to W$.

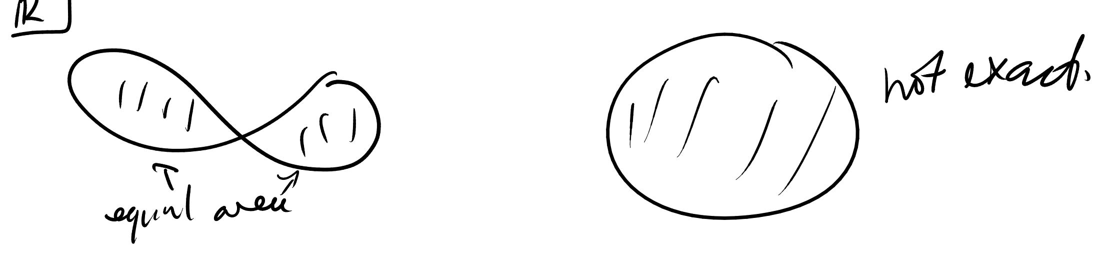
Formal solution: $\left(\begin{array}{ll} f: V \to W & \text{differentiable map,} \\ F: TV \to f^*TW & \text{isotropic monomorphism.} \end{array} \right)$

Genuine solution: $\left(\begin{array}{ll} f: V \to W & \text{s.t. } [f^*\theta] = 0 \\ F = df : TV \to f^*TW & \end{array} \right)$

The space of genuine solutions is homotopy equivalent to the space of formal solutions.

So existence of formal solution $\Rightarrow$ existence of genuine solution.

E.g. $\exists$ exact Lagrangian immersion of $S^n \to (\mathbb{R}^{2n}, \omega)$



But there is no exact Lagrangian embedding : self intersections are necessary.

Moral : Lagrangian embeddings are not governed by the h-principle.