

MATH 285 E1/F1 GRADED HOMEWORK SET 7
DUE MONDAY DECEMBER 8 IN LECTURE

This time, the homework has **just one part**. Please staple your homework together, and put your **name and section** on it. *Thank you!*

- (1) (15 points) Find the solution of following wave problem for vibrations in a string of length 10 (corresponding to a struck string).

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2} \\ u(0, t) = 0 \\ u(10, t) = 0 \\ u(x, 0) = 0 \\ \frac{\partial u}{\partial t}(x, 0) = x(10 - x) \end{cases}$$

- (2) (5 points) The function

$$u(x, t) = \sin(20t) \cos(5x)$$

solves a wave equation. Write $u(x, t)$ as a sum of a right-moving wave and a left-moving wave, and determine the speed of these waves. *Hint:* Scour the article “List of trigonometric identities” on Wikipedia for a relevant identity.

- (3) (15 points) Find the solution of the Laplace equation problem on the square $0 \leq x \leq 1$, $0 \leq y \leq 1$.

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \\ u(x, 0) = 1 \\ u(x, 1) = 0 \\ u(0, y) = 0 \\ u(1, y) = 0 \end{cases}$$