MATH 285 E1/F1 GRADED HOMEWORK SET 5
DUE WEDNESDAY NOVEMBER 5 IN LECTURE

This time, the homework has **just one part**. Please staple your homework together, and put your **name and section** on it. *Thank you!*

(1) (15 points, from [1, p. 190]) Expand x^3 and x in Fourier sine series valid when $-\pi < x < \pi$; and hence find the value of the sum of the series

$$\sin x - \frac{1}{2^3}\sin 2x + \frac{1}{3^3}\sin 3x - \frac{1}{4^3}\sin 4x + \cdots$$

for all values of x.[1]

(2) (5 points) Find the Fourier cosine series of the function $f(t) = 1 - t$ defined on the interval $0 < t < 1$.

(3) (10 points) Let $f(t)$ be the periodic function of period 2 defined on the interval $0 < t < 2$ by the formula $f(t) = t^2$. The Fourier series of this function is

$$\frac{4}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos n\pi t - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\pi t$$

(a) (5 points) Describe precisely the value of the sum of the Fourier series at every value of t, including at the points of discontinuity of the original function $f(t)$.

(b) (5 points) Suppose we differentiate the Fourier series term-by-term. Show that the resulting series does not converge (to anything, in particular not to $f'(t)$). *Hint:* try to plug $t = 1/2$ into the differentiated series.

(4) (5 points) Let $F(t)$ be the odd function of period 2π such that $F(t) = 1$ for $0 < t < \pi$ (this is a square wave). Consider the mass-spring system with $m = 1$, $k = 5$, subject to the driving force $F(t)$:

$$\frac{d^2x}{dt^2} + 5x = F(t)$$

Using Fourier series methods, find a steady periodic solution of this differential equation.

References

[1] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis*, fourth edition.

[1] According to Whittaker and Watson [1], this problem was on an exam at Jesus College, Cambridge, in 1902.