MATH 285 E1/F1 GRADED HOMEWORK SET 4
DUE FRIDAY OCTOBER 24 IN LECTURE

This time, the homework has **just one part**. Please staple your homework together, and put your **name and section** on it. *Thank you!*

(1) (10 points) Find the *general solution* of the differential equation

$$(D-3)y = xe^{2x} + e^{3x}$$

Hint: Use the annihilator method.

(2) (5 points) Consider the forced mass-spring oscillator with mass $m = 4$, no damping $c = 0$, and a spring constant k that is adjustable. Consider a driving forces that is the sum of two cosine functions

$$F(t) = \cos 2t + \cos 5t.$$

The differential equation for the displacement $x(t)$ is then

$$4x'' + kx = \cos 2t + \cos 5t.$$

For what values of k is it impossible to find a particular solution of the form $A\cos 2t + B\cos 5t$? (These are the values of k for which resonance occurs.)

(3) (5 points) Recall from the lecture the damped ($c > 0$) forced oscillator obeying the differential equation

$$mx'' + cx' + kx = F_0 \cos \omega t.$$

In the lecture, we derived a particular solution of this equation

$$x_p(t) = C\cos(\omega t - \alpha),$$

where

$$C = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}}, \quad \tan\alpha = \frac{c\omega}{k - m\omega^2}.$$

Here is something we did not talk about in the lecture: the solution $x_p(t)$ is called the *steady-state* solution, because any solution will asymptotically converge to it as $t \to \infty$. More precisely, if $x_1(t)$ is *any solution* of the same nonhomogeneous equation, then

$$\lim_{t\to\infty} [x_1(t) - x_p(t)] = 0.$$

Explain why this is true. You should use the description of the solutions of the homogeneous equation $mx'' + cx' + kx = 0$ on page 191 of the textbook. *Hint:* It is very important that $c > 0$.

(4) (5 points) Use the orthogonality properties of sine and cosine to compute the following integral

$$\int_{-\pi}^{\pi} (5\sin 2t - \cos 3t)(\cos 2t + 5\sin 6t + \sin 2t)\, dt$$