MATH 285 E1/F1 GRADED HOMEWORK SET 3
DUE WEDNESDAY OCTOBER 8 IN LECTURE

This time, the homework has **just one part**. Please staple your homework together, and put your **name and section** on it. *Thank you!*

(1) (5 points) Show that the functions e^{3x}, xe^{3x}, and x^2e^{3x} are linearly independent. That is, show that, if

$$c_1e^{3x} + c_2xe^{3x} + c_3x^2e^{3x} = 0 \text{ for all } x,$$

then it necessarily follows that $c_1 = c_2 = c_3 = 0$.

Hint: Get relations between the c_i's by plugging in $x = 0$ and/or differentiating the equation.

(2) (15 points) Consider the polynomial

$$\begin{aligned} p(r) = {} & r^{12} - 12r^{11} + 51r^{10} + r^9 - 1968r^8 + 19003r^7 \\ & - 106948r^6 + 440432r^5 - 1423168r^4 + 3448064r^3 \\ & - 6069248r^2 + 7606272r - 5160960. \end{aligned}$$

Find the general solution of the linear homogeneous equation

$$p(D)y = 0,$$

where as usual $D = \frac{d}{dx}$, and $p(D)$ denotes the constant coefficient differential operator obtained by substituting D for r in $p(r)$. We are looking for a general solution that is real (not complex) and which involves twelve arbitrary constants.

Hint: You should use the following factorization of $p(r)$:

$$p(r) = (r-3)^2(r-4)^3(r+7)(r^2-r+5)(r^2+16)^2$$

(3) (10 points) A body of mass $m = 5$ kg is attached to a spring with spring constant $k = 20$ kg/s^2. The body is suspended on an "air-hockey" table so that it is not subject to friction or gravity, but it is connected to a dashpot mechanism that lets us adjust the degree of damping, $c \geq 0$. The equation of motion is the usual one

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0.$$

Consider the following experimental procedure: move the body to an initial position $x(0) = 0$ m, and set it into motion with initial velocity $v(0) = 1$ m/s.

Your task is to determine the position function $x(t)$ in the following three cases:

(a) $c = 10$ kg/s,

(b) $c = 20$ kg/s,

(c) $c = 30$ kg/s.

It is perfectly fine to refer to section 3.4 of the textbook in your solution (that is, you don't have to rederive everything all over again, but you should mention what facts from the textbook you are using).

(4) (5 points) Find a particular solution of

$$y'' - 4y' + y = 3e^x.$$