NAME:

NetID:

MATH 285 E1/F1 Exam 3 (C) **November 14, 2014** **Instructor: Pascaleff**

INSTRUCTIONS:

- Do all work on these sheets.
- Show all work.
- The exam is 50 minutes.
- Do not discuss this exam with anyone until after 3:00 pm on Nov. 14, 2014.

Problem	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Orthogonality formulas

$$\int_{-L}^{L} \cos\frac{m\pi t}{L}\cos\frac{n\pi t}{L}\,dt = \begin{cases} 0, & m \neq n \\ L, & m = n \end{cases} \tag{1}$$

$$\int_{-L}^{L} \sin\frac{m\pi t}{L}\sin\frac{n\pi t}{L}\,dt = \begin{cases} 0, & m \neq n \\ L, & m = n \end{cases} \tag{2}$$

$$\int_{-L}^{L} \cos\frac{m\pi t}{L}\sin\frac{n\pi t}{L}\,dt = 0 \tag{3}$$

Some integral formulas

$$\int u\cos u\,du = u\sin u + \cos u + C \tag{4}$$

$$\int u\sin u\,du = -u\cos u + \sin u + C \tag{5}$$

1. (20 points) Find the general solution of the differential equation

$$y' - 4y = xe^{4x}$$

2. (20 points) Consider the forced oscillator with mass $m = 1$, spring constant $k = 7$, no damping $c = 0$, and forcing function $F(t)$:

$$F(t) = 2\sin 2t + \cos 4t + \cos 6t$$

Find a particular solution of the differential equation $mx'' + kx = F(t)$.

3. (a) (10 points) Suppose that a function $f(t)$ which is periodic of period 2π has the Fourier series

$$f(t) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+3} \sin nt$$

Use the orthogonality formulas to evaluate the integral

$$\int_{-\pi}^{\pi} f(t) \sin 2t \, dt$$

(b) (10 points) Let $g(t)$ be the function which is periodic of period 24, and which is defined on the interval $-12 \leq t < 12$ by the formula

$$g(t) = 4 + 2t + 7t^2$$

Set up, but do not evaluate, an integral expression for the coefficient of $\cos \frac{5\pi t}{12}$ in the Fourier series of $g(t)$ (also known as a_5 in our standard notation).

4. (a) (5 points) Consider the function which is periodic of period 2π defined on the interval $-\pi \le t < \pi$

$$f(t) = \begin{cases} 800, & -\pi \le t < 0 \\ 609250, & t = 0 \\ t, & 0 < t < \pi \end{cases}$$

If we take the Fourier series of $f(t)$, and put $t = 0$ in that series, what number does it converge to? Put another way, what is the sum of the Fourier series of $f(t)$ at $t = 0$? Explain your answer (briefly).

(b) (15 points) Consider the function defined by the Fourier series

$$g(t) = \sum_{n=1}^{\infty} 2e^{-3n} \sin n\pi t$$

Find a Fourier series expression for the antiderivative $\int g(t)\,dt$. You are *not* expected to address the question of convergence.

5. (20 points) Find the Fourier series of the periodic function of period 2 defined on the interval $-1 \leq t < 1$ by

$$f(t) = 3|t|, \quad -1 \leq t < 1$$

Hint: You should use the fact that $f(t)$ is an even function.

This page is for work that doesn't fit on other pages.