

NAME:

NetID:

MATH 285 E1/F1 Exam 3 (B)

November 14, 2014

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<p>INSTRUCTIONS:</p> <ul style="list-style-type: none">• Do all work on these sheets.• Show all work.• The exam is 50 minutes.• Do not discuss this exam with anyone until after 3:00 pm on Nov. 14, 2014.
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Problem	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

ORTHOGONALITY FORMULAS

$$\int_{-L}^L \cos \frac{m\pi t}{L} \cos \frac{n\pi t}{L} dt = \begin{cases} 0, & m \neq n \\ L, & m = n \end{cases} \quad (1)$$

$$\int_{-L}^L \sin \frac{m\pi t}{L} \sin \frac{n\pi t}{L} dt = \begin{cases} 0, & m \neq n \\ L, & m = n \end{cases} \quad (2)$$

$$\int_{-L}^L \cos \frac{m\pi t}{L} \sin \frac{n\pi t}{L} dt = 0 \quad (3)$$

SOME INTEGRAL FORMULAS

$$\int u \cos u \, du = u \sin u + \cos u + C \quad (4)$$

$$\int u \sin u \, du = -u \cos u + \sin u + C \quad (5)$$

1. (20 points) Find the general solution of the differential equation

$$y' - 5y = xe^{5x}$$

2. (20 points) Consider the forced oscillator with mass $m = 1$, spring constant $k = 5$, no damping $c = 0$, and forcing function $F(t)$:

$$F(t) = \cos 2t + \sin 4t + 3 \sin 6t$$

Find a particular solution of the differential equation $mx'' + kx = F(t)$.

3. (a) (10 points) Suppose that a function $f(t)$ which is periodic of period 2π has the Fourier series

$$f(t) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 1} \sin nt$$

Use the orthogonality formulas to evaluate the integral

$$\int_{-\pi}^{\pi} f(t) \sin 3t dt$$

- (b) (10 points) Let $g(t)$ be the function which is periodic of period 20, and which is defined on the interval $-10 \leq t < 10$ by the formula

$$g(t) = 7t^2 + 2t + 3$$

Set up, but do not evaluate, an integral expression for the coefficient of $\sin \frac{3\pi t}{10}$ in the Fourier series of $g(t)$ (also known as b_3 in our standard notation).

4. (a) (5 points) Consider the function which is periodic of period 2π defined on the interval $-\pi \leq t < \pi$

$$f(t) = \begin{cases} t, & -\pi \leq t < 0 \\ 609250, & t = 0 \\ 8, & 0 < t < \pi \end{cases}$$

If we take the Fourier series of $f(t)$, and put $t = 0$ in that series, what number does it converge to? Put another way, what is the sum of the Fourier series of $f(t)$ at $t = 0$? Explain your answer (briefly).

- (b) (15 points) Consider the function defined by the Fourier series

$$g(t) = \sum_{n=1}^{\infty} 4e^{-4n} \sin n\pi t$$

Find a Fourier series expression for the antiderivative $\int g(t) dt$. You are *not* expected to address the question of convergence.

5. (20 points) Find the Fourier series of the periodic function of period 2 defined on the interval $-1 \leq t < 1$ by

$$f(t) = 4|t|, \quad -1 \leq t < 1$$

Hint: You should use the fact that $f(t)$ is an even function.

This page is for work that doesn't fit on other pages.