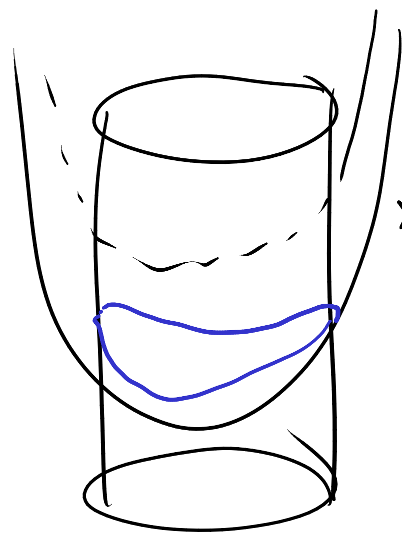
Review Final: May 10 9-12 noon

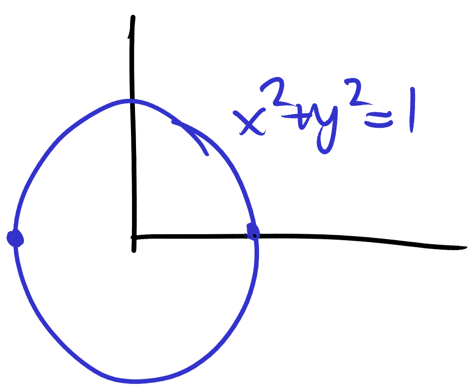
- Parametrization of curves of intersection.
- Change of coordinates.
- vector and scalar projection.
- graphing parametric curves.

Curve of intersection



Parametrize curve of intersection.

equation $z = x^2$ determines z from x
Forget z temporarily, project onto xy-plane.



$$\left.\begin{aligned} x &= t \\ y &= \pm\sqrt{1-t^2} \end{aligned}\right\} \text{only half circle}$$

$$\begin{aligned} x &= \cos t \\ y &= \sin t \end{aligned} \qquad \text{whole circle } 0 \le t \le 2\pi$$

Remember $z = x^2 = \cos^2 t$

Ans $\begin{cases} x = \cos t \\ y = \sin t \\ z = \cos^2 t \end{cases}$ $\vec{r}(t) = \cos t\,\vec{i} + \sin t\,\vec{j} + \cos^2 t\,\vec{k}$

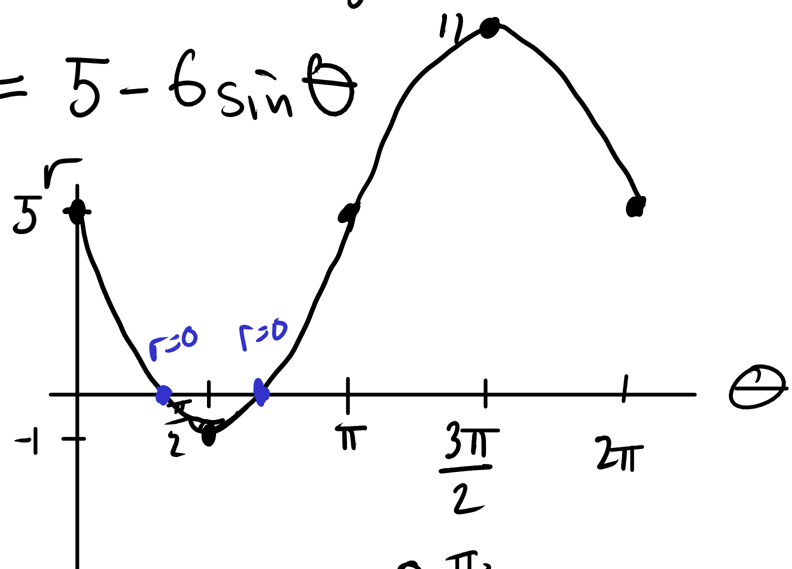
or $\left.\begin{array}{l} x = \sin t \\ y = \cos t \\ z = \sin^2 t \end{array}\right\}$ also correct. clockwise in xy-plane.

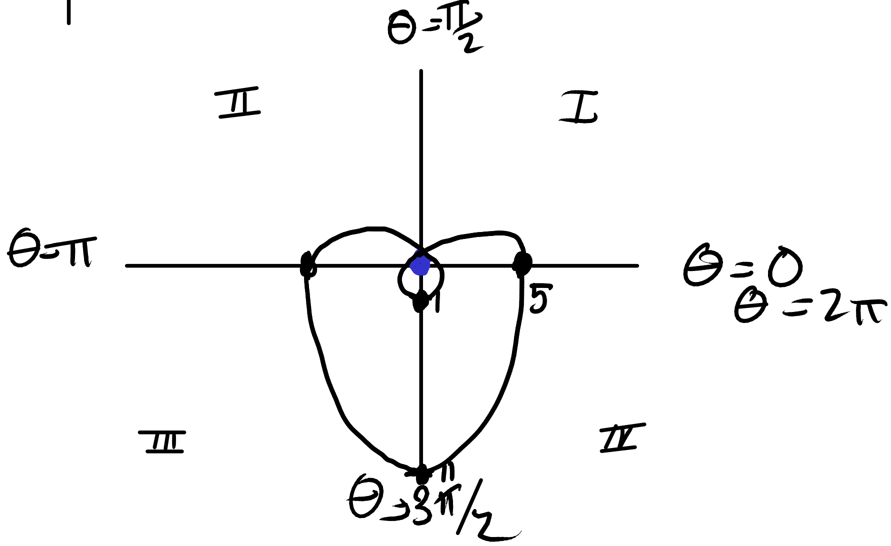
Many parametrizations.

Graphing polar curves $r = f(\theta)$

$$x = r\cos\theta = f(\theta)\cos\theta$$
$$y = r\sin\theta = f(\theta)\sin\theta$$

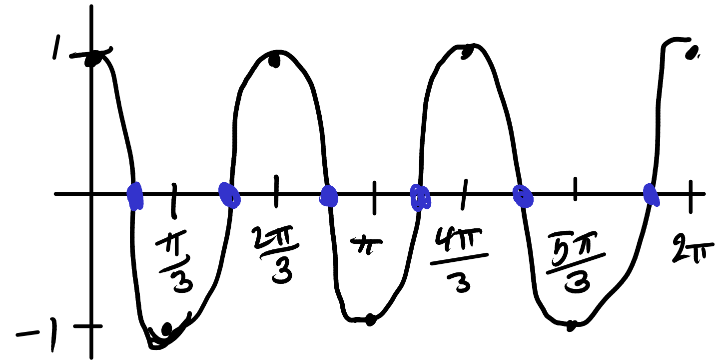
$r = 5 - 6\sin\theta$

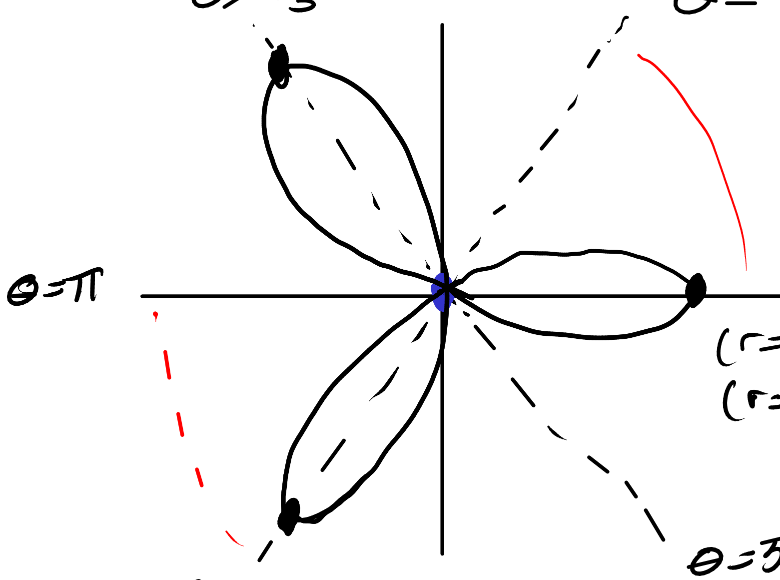




cardioid with a loop.

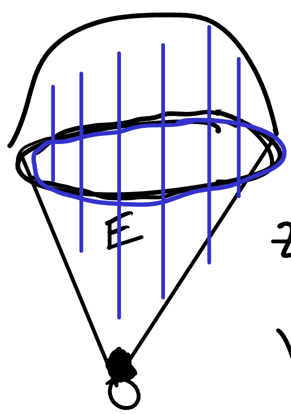
$r = \cos 3\theta$





closes up after angle π

Transformations / change of coordinates.



$x^2+y^2+z^2=1$ ← Top

$z = \sqrt{1-x^2-y^2}$

$z = \sqrt{x^2+y^2}$ ← bottom

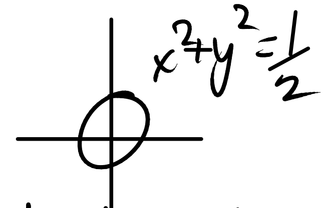
$$V = \iiint_E 1\, dV$$

$x^2+y^2+\left(\sqrt{x^2+y^2}\right)^2 = 1 \qquad 2(x^2+y^2) = 1$

$\{ x^2+y^2 = \frac{1}{2}$ circle of radius $\frac{1}{\sqrt{2}}$ sitting at $z = \frac{1}{\sqrt{2}} \}$

$$V = \iiint_E dV = \iint_D \left[\int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} dz \right] dA$$

project to xy-plane : just get circle



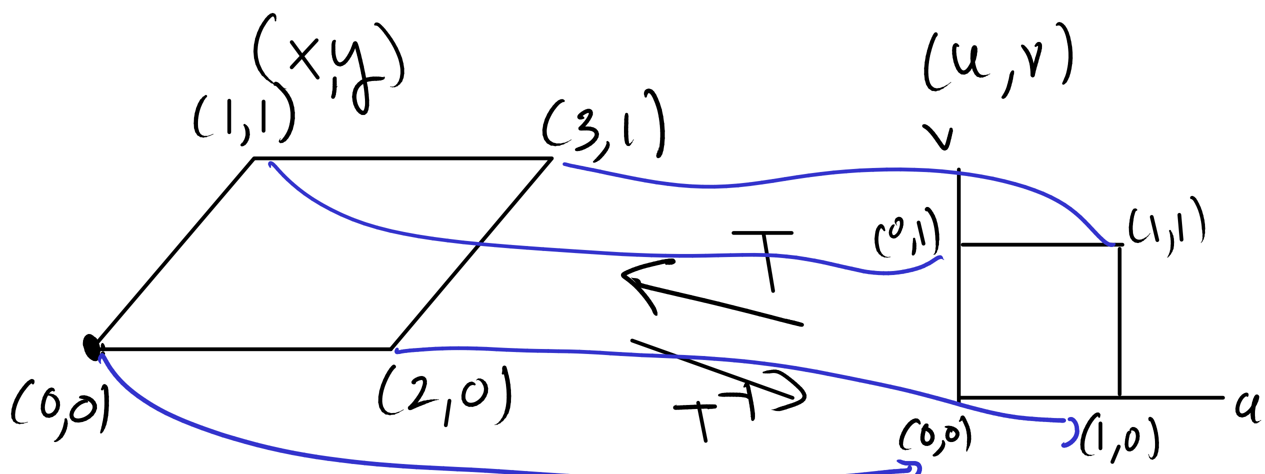
· Can use polar coordinates for this double integral

$$V = \iint_D \left[\sqrt{1-x^2-y^2} - \sqrt{x^2+y^2} \right] dA$$

Polar coords $D = \left\{ 0 \le r \le \frac{1}{\sqrt{2}},\ 0 \le \theta \le 2\pi \right\}$

$x^2+y^2 = r^2$ $\quad dA = r\,dr\,d\theta$

$$V = \int_0^{2\pi} \int_0^{1/\sqrt{2}} \left[\sqrt{1-r^2} - r \right] r\,dr\,d\theta$$



$$I = \iint_{\text{in } (x,y)} f\, dA = \iint f \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\, dv$$

$$dA = dx\,dy = \left| \frac{\partial(x,y)}{\partial(\cancel{u},\cancel{v})} \right| d\cancel{u}\,dv$$

$x = g(u,v)$
$y = h(u,v)$

Transformation: Can take $(x,y) = T(u,v)$ to be <u>linear</u>.
(because we're trying to transform a rectangle into a parallelogram)

$$x = au + bv + e$$
$$y = cu + dv + f$$

	(u,v)	(x,y)
1	$(0,0)$	$(0,0)$
2	$(1,0)$	$(2,0)$
3	$(1,1)$	$(3,1)$
4	$(0,1)$	$(1,1)$

$$1 \begin{cases} 0 = a0 + b0 + e \\ 0 = c0 + d0 + f \end{cases}$$

$e = 0$
$f = 0$

$$2 \begin{cases} 2 = a1 + b0 & a = 2 \\ 0 = c1 + d0 & c = 0 \end{cases}$$

$$3 \begin{cases} 3 = 2\cdot 1 + b1 & 3 = 2 + b & b = 1 \\ 1 = d1 & 1 = d & d = 1 \end{cases}$$

$$x = 2u + v \qquad x = 2u + v$$
$$y = v \qquad y = v$$

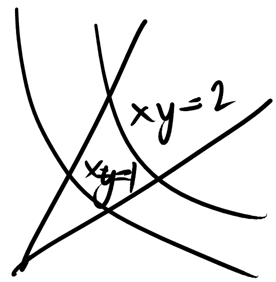
$$4 \begin{cases} 1 = 2\cdot 0 + 1 \\ 1 = 1 \end{cases} \checkmark$$

$$\frac{\partial x}{\partial u} = 2 \quad \frac{\partial x}{\partial v} = 1$$
$$\frac{\partial y}{\partial u} = 0 \quad \frac{\partial y}{\partial v} = 1$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2$$

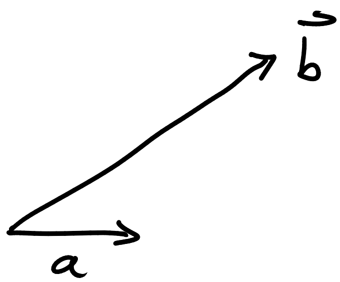
$$\iint f\,dA = \int_0^1\int_0^1 f\; 2\,du\,dv$$

Tip:

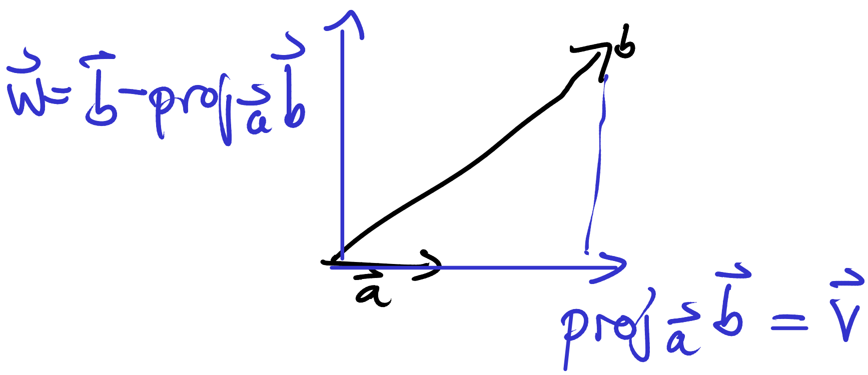


consider using $u = xy$ as one of the coordinates.

Vector and scalar projection.



want to decompose $\vec{b}$ into a sum of two vectors, one parallel to $\vec{a}$, and one perpendicular to $\vec{a}$



$$\vec{b} = \vec{v} + \vec{w}$$
$$\vec{v} \parallel \vec{a}$$
$$\vec{w} \perp \vec{a}$$

Find a formula for $\vec{v}$

$$\vec{a}\cdot\vec{b} = \vec{a}\cdot(\vec{v}+\vec{w}) = \vec{a}\cdot\vec{v} + \vec{a}\cdot\vec{w}$$

$$\left\{\begin{array}{l} \vec{v} \parallel \vec{a} \Rightarrow \boxed{\vec{v} = k\vec{a}} \\ \vec{w} \perp \vec{a} \Rightarrow \vec{a}\cdot\vec{w} = 0 \end{array}\right\} = \vec{a}\cdot k\vec{a} + 0$$

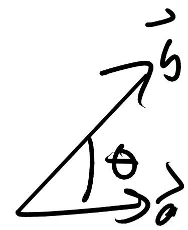
$$= k\,\vec{a}\cdot\vec{a}$$

$$k = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}} \qquad \text{proj}_{\vec{a}}\vec{b} = \vec{V} = k\vec{a} = \left(\frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\right)\vec{a}$$

$$\text{proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\right)\vec{a} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|}$$

$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$ = unit vector in direction of $\vec{a}$

$$\text{proj}_{\vec{a}}\vec{b} = \underbrace{\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)}_{\text{comp}_{\vec{a}}\vec{b}}\hat{a}$$



$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = |\vec{b}|\cos\theta$$

$\text{proj}_{\vec{a}}\vec{b} = \left(\text{comp}_{\vec{a}}\vec{b}\right)\hat{a}$ where $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

$$\text{comp}_{\vec{a}}\vec{b} = \hat{a}\cdot\vec{b} = \text{comp}_{\hat{a}}\vec{b}$$