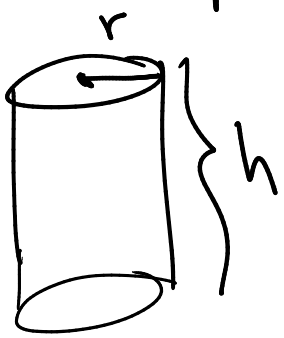
Functions of several variables (Ch 14)

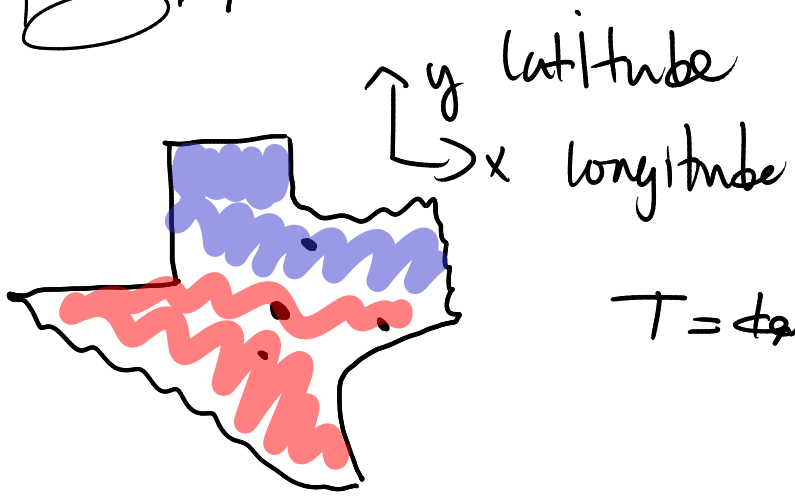
Two (or more) independent variables $x, y, \ldots$

one dependent variable $z = f(x, y)$



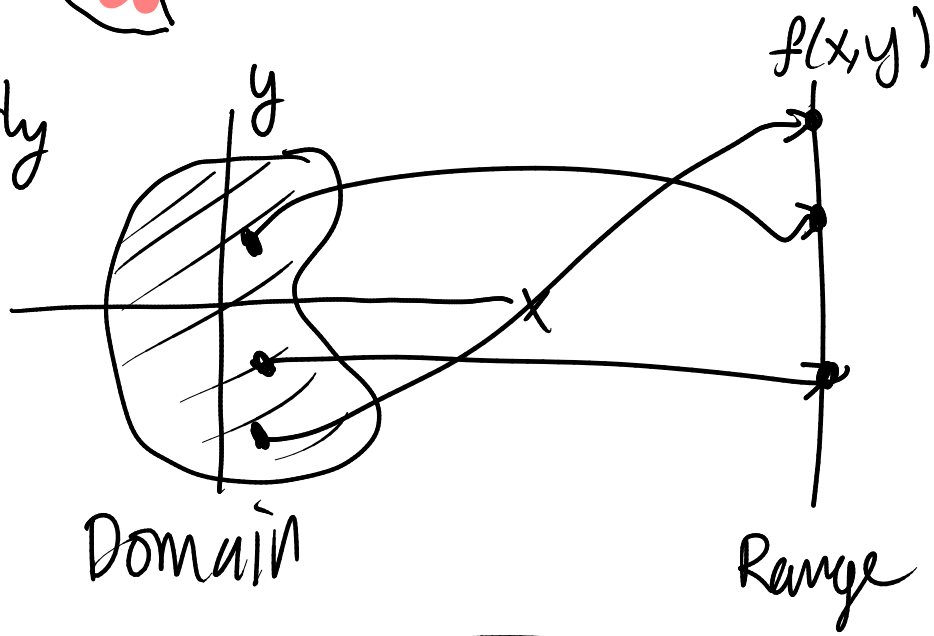
$$V = \pi r^2 h$$

$$V = V(r, h)$$



T = temperature $= T(x, y)$

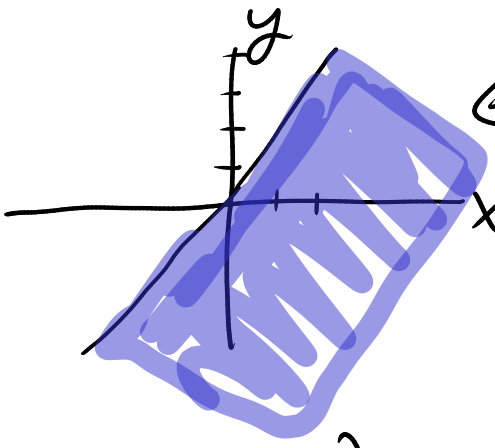
Abstractly



Find domain for $f(x, y) = \sqrt{2x - y}$

Need $2x - y \geq 0$

$y \leq 2x$



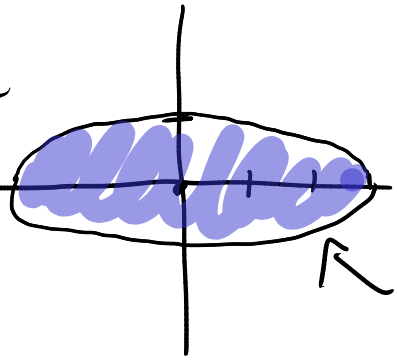
← this is the domain

$f(x,y) = \ln(9 - x^2 - 9y^2) \Rightarrow 9 - x^2 - 9y^2 > 0$

$x^2 + 9y^2 < 9 \qquad \frac{x^2}{9} + y^2 < 1$

Ellipse

Not including the ellipse itself



$f(x,y)$ goes to $-\infty$ at boundary.

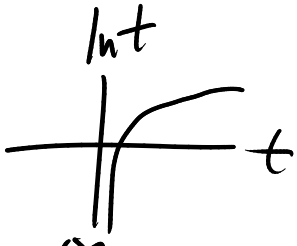


Table of values

$f(x,y) = x + y$

y \ x	-3	-2	-1	0	1	2	3
3	0	1	2	3	4	5	6
2	-1	0	1	2	3	4	5
1	-2	-1	0	1	2	3	4
0	-3	-2	-1	0	1	2	3
-1							
-2							
-3							

Graphs $z = f(x,y)$ is a surface in 3D.

$f(x,y) = 1 - x - 2y$

graph $z = 1 - x - 2y$

$x + 2y + z = 1$ plane

$f(x,y) = \sqrt{9 - x^2 - y^2}$

$z = \sqrt{9 - x^2 - y^2}$

$z^2 = 9 - x^2 - y^2$

$x^2 + y^2 + z^2 = 9$ sphere of radius 3.

$f(x,y) = x^2 + y^2$

$z = x^2 + y^2$ Elliptic paraboloid

$f(x,y) = x^2 - y^2$ hyperbolic paraboloid

Contour plot & level curves

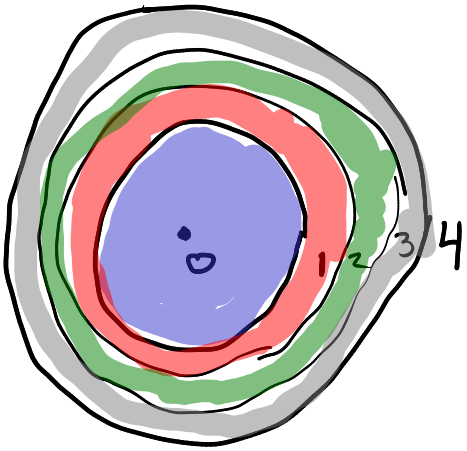
level curve = set where $f(x,y)$ takes a particular value

$f(x,y) = C$ constant

$f(x,y) = x^2+y^2$ Level curve $f(x,y)=c$

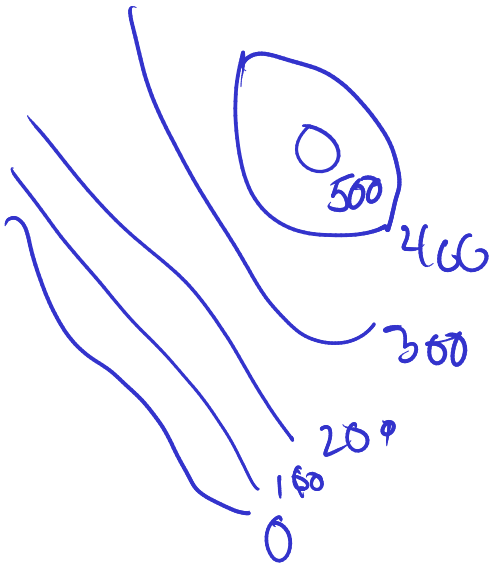
$x^2+y^2=c$

circle of radius $\sqrt{c}$



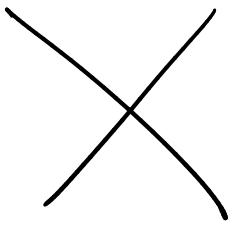
Topographic map:

= contour plot of elevation function.

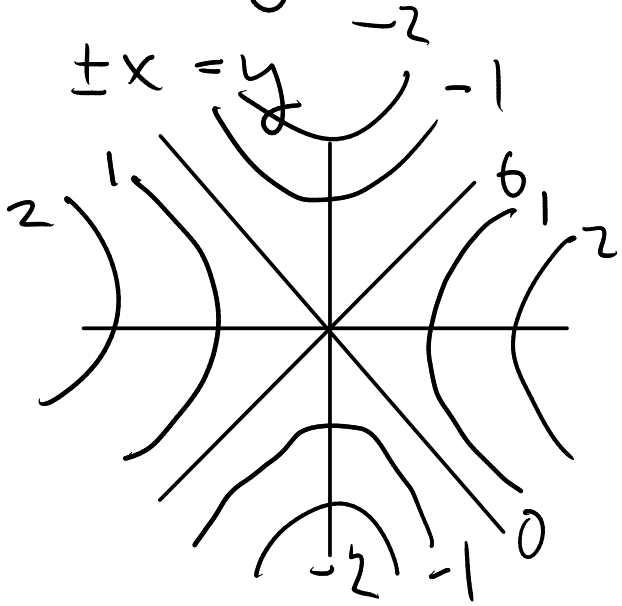


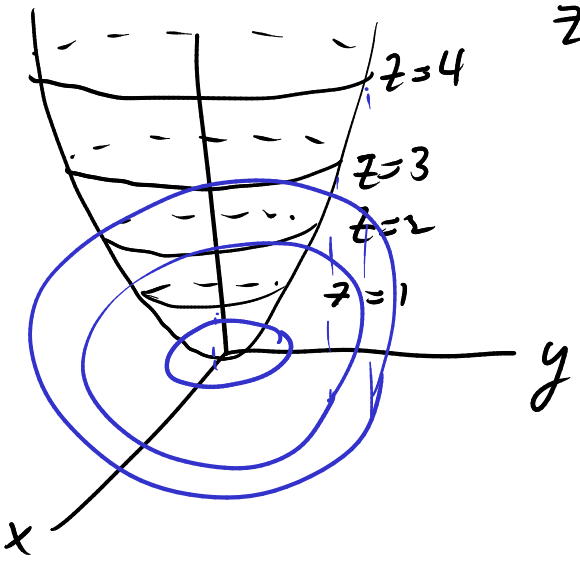
$f(x,y) = x^2-y^2$

$x^2-y^2=0 \Rightarrow$



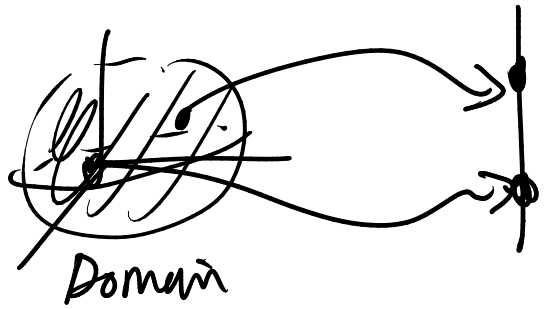
$\pm x = y$

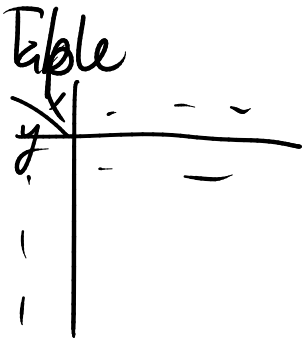




$z = x^2 + y^2$

More than two variables $f(x,y,z)$





Graph $w = f(x,y,z)$ exists in 4 dimensions. ☹

Contourplot $f(x,y,z) = C$ level surface

$f(x,y,z) = x^2 + y^2 + z^2$

$x^2 + y^2 + z^2 = C$ sphere of radius $\sqrt{C}$

More than 3 variables $x_1, x_2, x_3, x_4, \ldots, x_n$

$f(x_1, x_2, \ldots, x_n)$

Physics: n-body problem $\Rightarrow$ $3n$ variables

Limits of functions of 2 variables

$$f(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2} \qquad g(x,y) = \frac{x^2-y^2}{x^2+y^2}$$

What is limit as $(x,y) \to (0,0)$

$$\lim_{x\to 0} f(x,y) = \frac{\sin(y^2)}{y^2} \qquad \lim_{y\to 0} \frac{\sin(y^2)}{y^2} = 1$$

$$\lim_{y\to 0} \lim_{x\to 0} \frac{\sin(x^2+y^2)}{x^2+y} = 1$$

$$\lim_{y\to 0} \lim_{x\to 0} \frac{x^2-y^2}{x^2+y^2} = \lim_{y\to 0} \frac{-y^2}{y^2} = -1$$

$$\lim_{x\to 0} \lim_{y\to 0} \frac{x^2-y^2}{x^2+y^2} = \lim_{x\to 0} \frac{x^2}{x^2} = 1$$

does not agree

$\Rightarrow$ the limit doesn't exist.