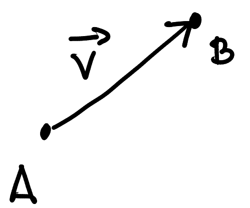
Vectors

A vector is an geometric object that has a magnitude (length) and a direction, but not necessarily a definite position.

Uses: displacement, velocity, acceleration, force, current, ...

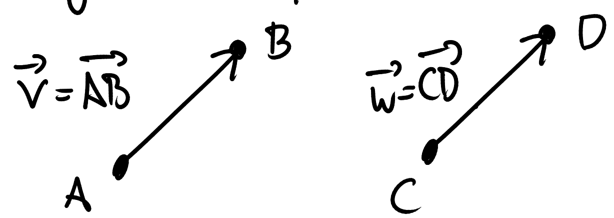
Displacement vector = vector from one point to another



$\vec{v} = \overrightarrow{AB}$

tail at A
tip at B.

Vectors are different from points: if we shift a vector we get an equivalent vector

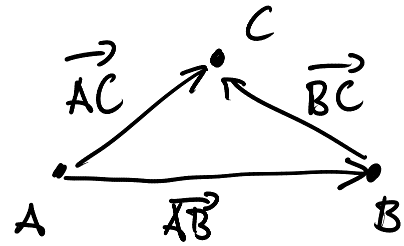


we write $\vec{v} = \vec{w}$

The vectors are the same because they have the same magnitude and direction, even though they are located at different points.

Operations on vectors.

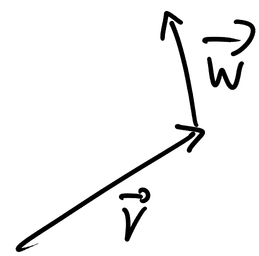
Addition:



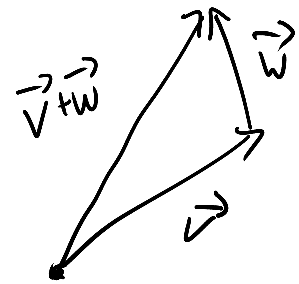
we write
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

This shows us how to add two vectors $\vec{v}$ and $\vec{w}$ (geometrically)

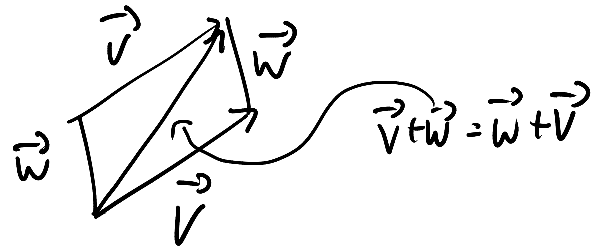
1. Place tail of $\vec{w}$ at tip of $\vec{v}$



2. Draw third side of triangle this is $\vec{v}+\vec{w}$



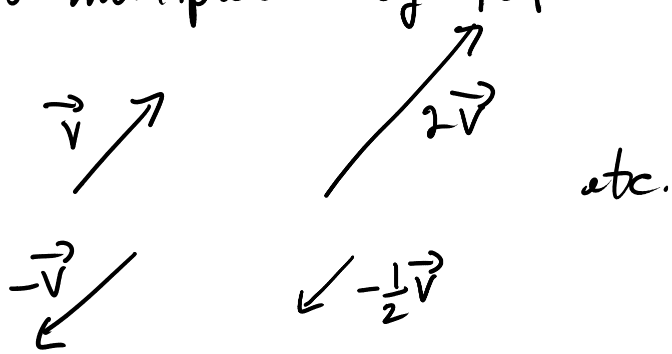
Could also use other order, get parallelogram



Scalar multiplication: c a number $\vec{v}$ a vector.

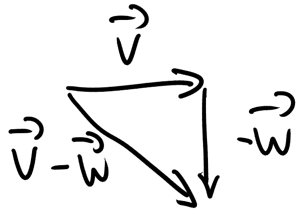
if $c>0$, $c\vec{v}$ has same direction as $\vec{v}$, but magnitude multiplied by c

if $c<0$, $c\vec{v}$ has opposite direction to $\vec{v}$, magnitude is multiplied by $|c|$



How to subtract vectors?

$$\vec{v} - \vec{w} = \vec{v} + (-\vec{w})$$

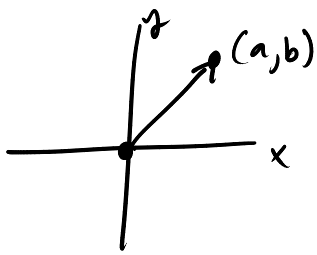


Components of a vector

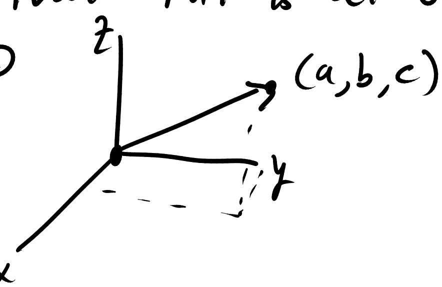
(representing a vector in terms of a coordinate system)

1. Place vector $\vec{v}$ so that tail is at origin

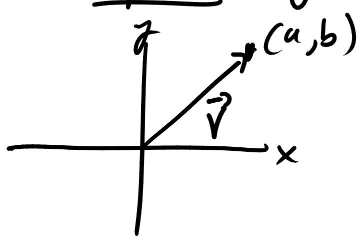
2D

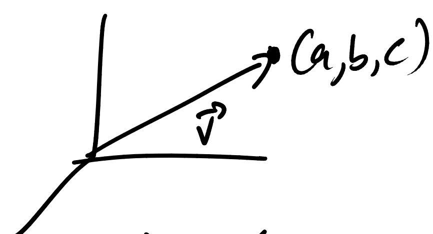


3D



2. The components of $\vec{v}$ are the coordinates of the tip.



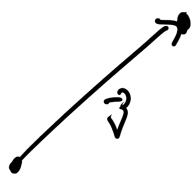


$$\vec{v} = \langle a, b \rangle \qquad \vec{v} = \langle a, b, c \rangle$$

- We use angle brackets $\langle , \rangle$ for components of a vector and parentheses $(,)$ for the coordinates of a point.

If we have a vector between two points

$B = (x_2, y_2, z_2)$



$A = (x_1, y_1, z_1)$

$$\vec{v} = \langle x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1 \rangle$$

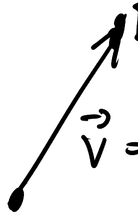
"tip minus tail"

Magnitude / length of vector : just use distance formula

$$\vec{v} = \langle v_1, v_2, v_3 \rangle \qquad |\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

magnitude / length

B



A

$$\vec{v} = \langle x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1 \rangle$$

$$|\vec{v}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2} = \text{distance from A to B.}$$

very easy to add / scalar multiply in components.
Just add / multiply one component at a time

$$\vec{v} = \langle v_1, v_2, v_3 \rangle \qquad \vec{w} = \langle w_1, w_2, w_3 \rangle$$

$$\vec{v} + \vec{w} = \langle v_1 + w_1,\ v_2 + w_2,\ v_3 + w_3 \rangle$$

$$c\vec{v} = \langle cv_1, cv_2, cv_3 \rangle$$

Basis vectors: Another way to notate components of a vector.

Define $\vec{i} = \langle 1,0,0\rangle$ $\quad \vec{j} = \langle 0,1,0\rangle$ $\quad \vec{k} = \langle 0,0,1\rangle$

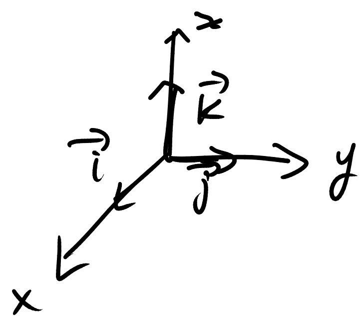
Consider $\vec{v} = \langle 5,2,-1\rangle$ can write $\vec{v}$ in terms of $\vec{i}, \vec{j}, \vec{k}$

$$5\vec{i} + 2\vec{j} - \vec{k} = 5\langle 1,0,0\rangle + 2\langle 0,1,0\rangle - \langle 0,0,1\rangle$$

$$= \langle 5,0,0\rangle + \langle 0,2,0\rangle + \langle 0,0,-1\rangle$$
$$= \langle 5,2,-1\rangle = \vec{v}$$

in general $\vec{v} = \langle v_1, v_2, v_3\rangle = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$

The <u>components</u> of $\vec{v}$ are the coefficients of $\vec{i}, \vec{j}, \vec{k}$



(in 2D just use $\vec{i}$ and $\vec{j}$)

<u>Unit vectors</u> A <u>unit vector</u> has magnitude 1 (and any direction)

$\vec{i}$ $\vec{j}$ and $\vec{k}$ are unit vectors. Given any vector $\vec{v}$, $\frac{1}{|\vec{v}|}\vec{v}$ is a unit vector with the same direction as $\vec{v}$

eg $\vec{v} = \langle 5,2,-1\rangle$ $\quad |\vec{v}| = \sqrt{5^2 + 2^2 + (-1)^2} = \sqrt{30}$

$\frac{1}{|\vec{v}|}\vec{v} = \left\langle \frac{5}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{-1}{\sqrt{30}} \right\rangle$ is a unit vector in same direction as $\vec{v}$.