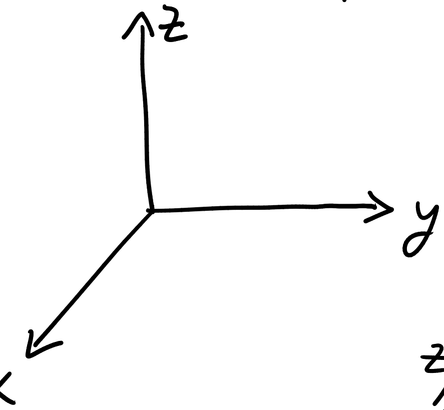
Three dimensional coordinate systems

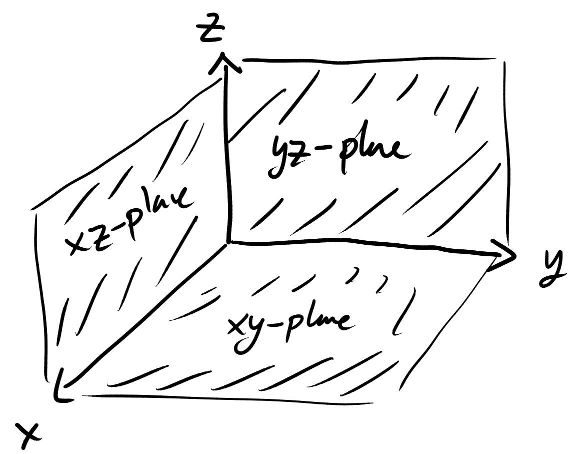
Just as two numbers (x,y) may be used to describe points in the (two-dimensional) plane, we use three numbers (x,y,z) to describe points in three-dimensional space.

Axes

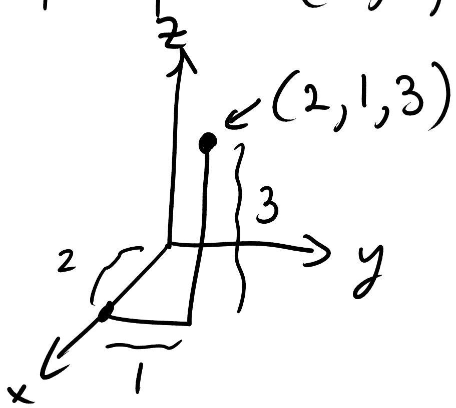


· imagine x-axis as coming out toward the viewer.

Coordinate planes
xy-plane = "floor"
xz & yz planes = "walls"



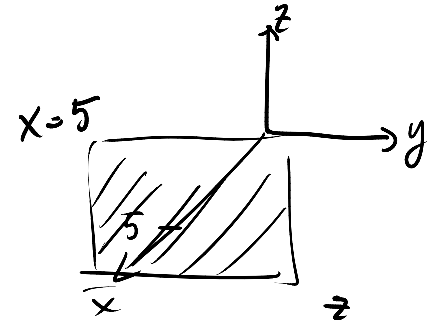
To plot point (a,b,c) go
a units along x-axis
b units parallel to y-axis
c units parallel to z-axis



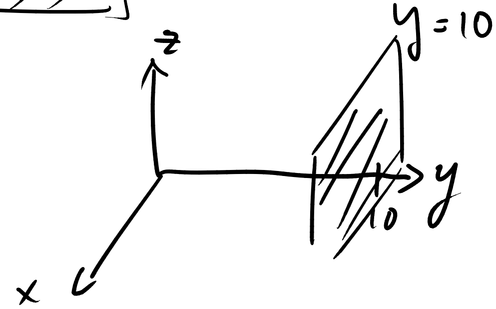
for $P = (a,b,c)$: $|a|$ = distance from P to yz-plane
$|b|$ = distance from P to xz-plane
$|c|$ = distance from P to xy-plane

Basic planes:

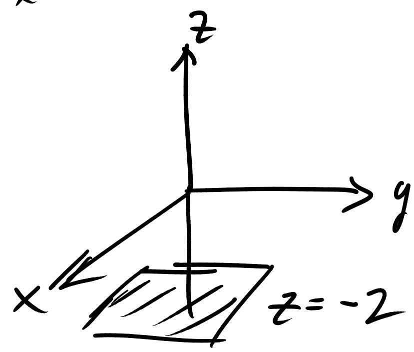
$x = 5$ defines a plane parallel to yz-plane



$y = 10$ is a plane parallel to xz-plane



$z = -2$ is a plane parallel to xy-plane



Observe that, in 3-dimensional space, the graph of a single equation such as $y=10$ is a 2-dimensional surface, not a curve.

Distances: distance between two points

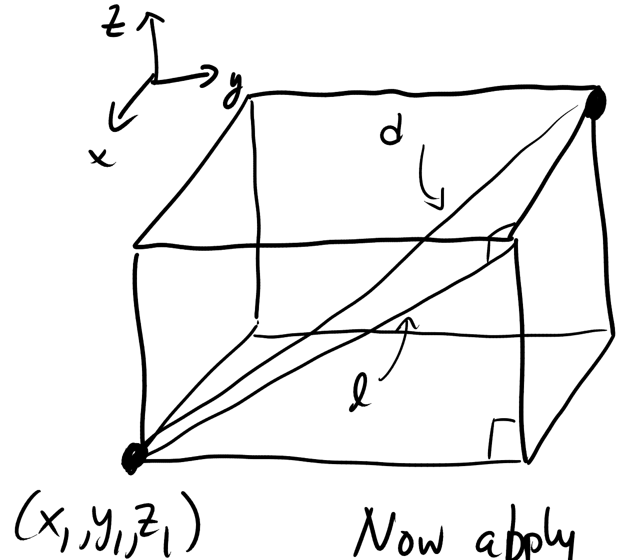
First point P_1 has coordinates (x_1, y_1, z_1)

second point P_2 has coordinates (x_2, y_2, z_2)

Then the distance between P_1 and P_2 is

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

We can see this using a rectangular prism with P_1 and P_2 at opposite vertices:



diagonal of front face

$$\ell = \sqrt{(y_2-y_1)^2 + (z_2-z_1)^2}$$

(by Pythagoren theorem)

Now apply pythagorean theorem again

$$(x_2-x_1)^2 + \ell^2 = d^2$$

$$(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2 = d^2$$

take square root of both sides $\to$ done.

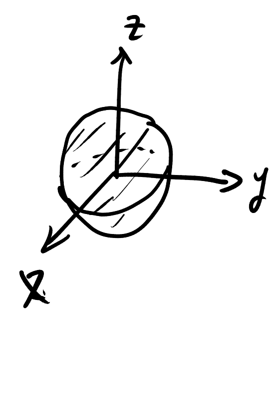
Spheres:

The set of points at a given distance from a fixed point is a sphere:

e.g. points at distance 2 from origin = sphere of radius 2

$$\sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2} = 2$$

$$x^2 + y^2 + z^2 = 2^2$$



Sphere centered at (x_0, y_0, z_0), radius r

$$\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} = r$$

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

Example: find center and radius of sphere given by equation

$$x^2+y^2+z^2-2x-4y+8z=15$$

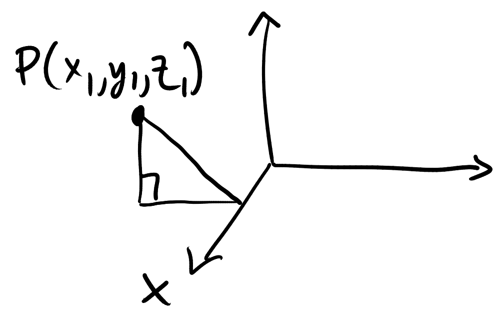
Complete the square:

$$x^2-2x=(x-1)^2-1$$
$$y^2-4x=(y-2)^2-4$$
$$z^2+8z=(z+4)^2-16$$

$$(x-1)^2-1+(y-2)^2-4+(z+4)^2-16=15$$
$$(x-1)^2+(y-2)^2+(z+4)^2=15+1+4+16=36$$
$$(x-1)^2+(y-2)^2+(z+4)^2=6^2$$

Center $=(1,2,-4)$ Radius $=6$

Distance from a point to an axis:



The distance from the point (x_1,y_1,z_1) to the x-axis is

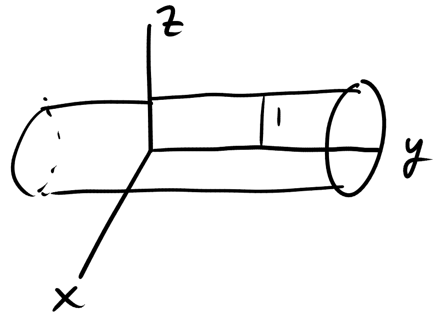
$$\sqrt{y_1^2+z_1^2}$$

Distance to y-axis $=\sqrt{x_1^2+z_1^2}$

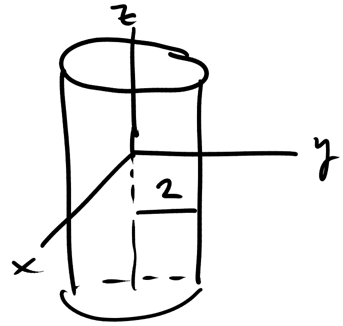
Distance to z-axis $=\sqrt{x_1^2+y_1^2}$

Set of points at given distance from an axis is a Cylinder

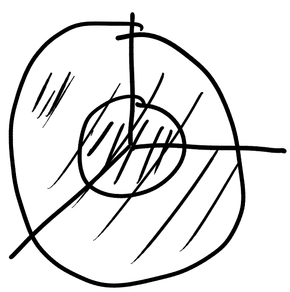
$$x^2+z^2=1$$



$x^2+y^2=4$



Example: write inequalities that define the set of points between the sphere of radius 2 and the sphere of radius 6, both centered at the origin. (including the points on the spheres)



$2 \le$ distance from origin ≤ 6

$2 \le \sqrt{x^2+y^2+z^2} \le 6$

$4 \le x^2+y^2+z^2 \le 36$