Exam Thursday 10/31 In Class

Covers up through lecture 16

Headed towards integration

Antiderivatives: opposite of derivative

two functions f and F

$F(x)$ is an antiderivative of $f(x)$

means $f(x)$ is the derivative of $F(x)$

Ex $f(x) = x^2$. what is an antiderivative of $f(x)$?

Try $F(x) = x^3$ $F'(x) = 3x^2$ wrong

Try $\boxed{F(x) = \frac{x^3}{3}}$ $F'(x) = \frac{1}{3}(3x^2) = x^2$ right ✓

Try $\boxed{F(x) = \frac{x^3}{3} + 5}$ $F'(x) = x^2$ also right ✓

If $F(x)$ is an antiderivative of $f(x)$

then $F(x) + c$ (c a constant) is also an

antiderivative of $f(x)$

If $F_1(x)$ and $F_2(x)$ are both antiderivatives of $f(x)$ (on a given interval)

then $F_1(x) = F_2(x) + C$ for some constant C (Any two antiderivatives differ by a constant)

Reason: $F_1(x)$ and $F_2(x)$ both antider. of $f(x)$

$$\Updownarrow$$

$$F_1'(x) = f(x) \qquad F_2'(x) = f(x)$$

$$\Downarrow$$

$$F_1'(x) = F_2'(x)$$

$$\Updownarrow$$

$$F_1'(x) - F_2'(x) = 0$$

$$\Updownarrow$$

$$\frac{d}{dx}\left(F_1(x) - F_2(x)\right) = 0$$

$$\Downarrow$$

$F_1(x) - F_2(x)$ is constant on any interval

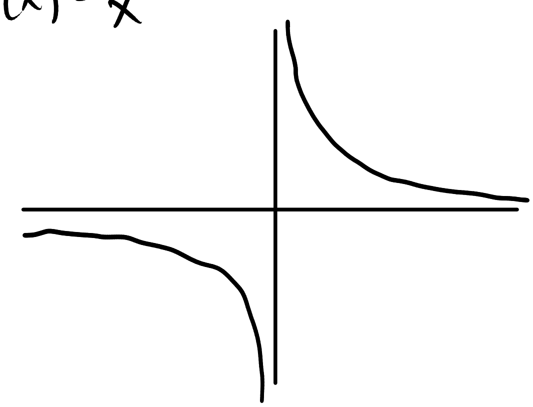
$$F_1(x) - F_2(x) = C$$

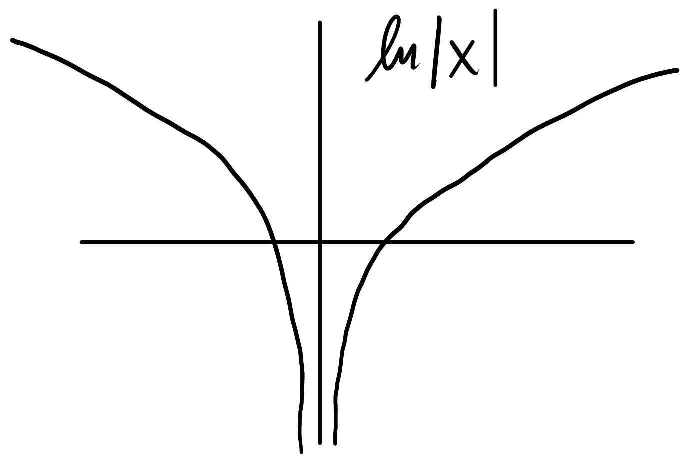
$$F_1(x) = F_2(x) + C$$

Find antiderivative

$f(x)$	an antiderivative $F(x)$	all antiderivatives
$\sin x$	$-\cos x$	$-\cos x + C$
$\cos x$	$\sin x$	$\sin x + C$
x^2	$\frac{x^3}{3}$	$\frac{x^3}{3} + C$
x^3	$\frac{x^4}{4}$	$\frac{x^4}{4} + C$
x^n	$\frac{x^{n+1}}{n+1}$	$\frac{x^{n+1}}{n+1} + C$
$(x>0)$ $x^{-1} = \frac{1}{x}$	$\ln x$	$\ln x + C$
$(x<0)$ $x^{-1} = \frac{1}{x}$	$\ln(-x)$	$\ln(-x) + C$

$f(x) = \frac{1}{x}$





$f(x)$	$F(x)$	$F(x)+C$
$\sec^2 x$	$\tan x$	$\tan x + C$
e^x	e^x	$e^x + C$
$\frac{1}{1+x^2}$	$\tan^{-1} x$	$\tan^{-1} x + C$

$$f(x) = 7x^{2/5} + 8x^{-4/5}$$

$$\frac{d}{dx}\left(\frac{x^{7/5}}{7/5}\right) = x^{2/5}$$

$$\frac{d}{dx}\left(7\,\frac{x^{7/5}}{7/5}\right) = 7x^{2/5}$$

$$\frac{d}{dx}\left(\frac{x^{1/5}}{1/5}\right) = x^{-4/5}$$

$$\frac{d}{dx}\left(8\,\frac{x^{1/5}}{1/5}\right) = 8x^{-4/5}$$

$$\frac{d}{dx}\left(7\,\frac{x^{7/5}}{7/5} + 8\,\frac{x^{1/5}}{1/5}\right) = 7x^{2/5} + 8x^{-4/5}$$

$$F(x) = 7\,\frac{x^{7/5}}{7/5} + 8\,\frac{x^{1/5}}{1/5} = 7\,\frac{5}{7}\,x^{7/5} + 8\,\frac{5}{1}\,x^{1/5}$$

$$F(x) = 5x^{7/5} + 40x^{1/5}$$ an antiderivative

Any antiderivative is $5x^{7/5} + 40x^{1/5} + C$.

$$f(x) = \sqrt{2}$$

$$\frac{x^{3/2}}{3/2} \longrightarrow x^{1/2}$$

$$F(x) = \sqrt{2} \cdot x$$

general $F(x) = \sqrt{2}x + C$

$$f(x) = \frac{1+t+t^2}{\sqrt{t}} = t^{-1/2}(1+t+t^2)$$

$$= t^{-1/2} + t^{1/2} + t^{3/2}$$

$$F(x) = \frac{t^{1/2}}{1/2} + \frac{t^{3/2}}{3/2} + \frac{t^{5/2}}{5/2} + C$$

$$f(x) = 2\sin x \cos x = \sin 2x$$

$$F(x) = -\frac{\cos(2x)}{2}$$ an antiderivative

$G(x) = \sin^2 x \qquad G'(x) = 2\sin x \cos x = f(x)$

Conclude that $-\dfrac{\cos(2x)}{2}$ and $\sin^2 x$

differ by a constant.

<u>Prob</u>: Use trig identities to figure out what the constant is.

Reconstruction of a function from its derivative and initial data.

(Differential Equations)

Kinematics : position velocity and acceleration.

Theory : $F=ma$, tells you what a = acceleration is.

$$\begin{array}{lll} & \text{position} & s(t) \\ \text{deriv.} & \text{velocity} & v(t) = s'(t) \\ \text{deriv.} & \text{acceleration} & a(t) = v'(t) = s''(t) \end{array}$$

antideriv.

antideriv.

constant acceleration under gravity $s \uparrow$ $\downarrow g = 9.8 m/s^2$

$a(t) = -g$ constant

$v(t) = -gt + C$

C, D constants

$s(t) = -g\dfrac{t^2}{2} + Ct + D$

# of constants = # of times you antidifferentiation

$v(0) = -g(0) + C = 0 + C = C$

$s(0) = -g\dfrac{0^2}{2} + C\cdot 0 + D = D$

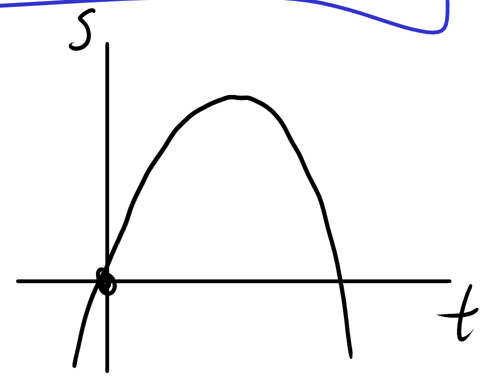
C = initial velocity D = initial position.

e.g. initial data

$\boxed{\text{initial position} = 0 \qquad \text{initial velocity} = 10}$

$$s(t) = -g\frac{t^2}{2} + 10t + 0$$

$$= -9.8\frac{t^2}{2} + 10t$$



Know : $f'(x) = 1 + 3\sqrt{x}$ ← differential equation

$f(4) = 25$ ← initial data

What is $f(x)$?

1) Find antiderivative of $f'(x)$

2) use the the initial data to fix the constant

Antiderivative $f(x) = x + 3\dfrac{x^{3/2}}{3/2} + C$

What is C?

$$25 = f(4) = 4 + 3\frac{(4)^{3/2}}{3/2} + C$$

$$25 = 4 + 3\tfrac{2}{3}\cdot 2^3 + C$$

$$25 = 4 + 16 + C \Rightarrow 25 = 20 + C \quad \boxed{C = 5}$$

full answer $f(x) = x + 3\dfrac{x^{3/2}}{3/2} + 5$

$$f(x) = x + 2x^{3/2} + 5$$

There is a unique answer!

$f''(x) = 8x^3 + 5$, $f(1) = 0$, $f'(1) = 8$

What is $f(x)$?

deriv f antideriv

f'

deriv f'' antideriv

$$f'(x) = 8\frac{x^4}{4} + 5x + C = 2x^4 + 5x + C$$

$$8 = f'(1) = 2(1)^4 + 5(1) + C$$

$$8 = 2 + 5 + C = 7 + C \qquad C = 1$$

$$f'(x) = 2x^4 + 5x + 1$$

$$f(x) = 2\frac{x^5}{5} + 5\frac{x^2}{2} + x + D$$

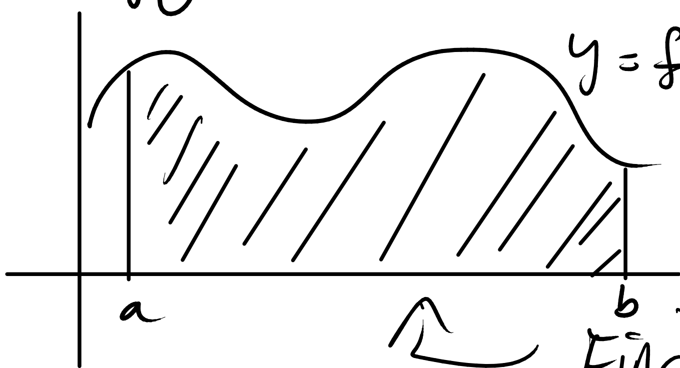
$$0 = f(1) = 2\frac{(1)^5}{5} + \frac{5(1)^2}{2} + 1 + D$$

$$0 = \frac{2}{5} + \frac{5}{2} + 1 + D \qquad D = -\frac{39}{10}$$

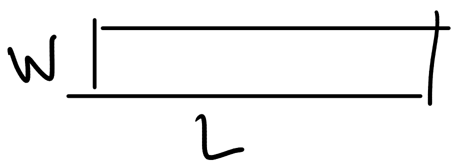
$$0 = \frac{4}{10} + \frac{25}{10} + \frac{10}{10} + D$$

$$f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x - \frac{39}{10}$$

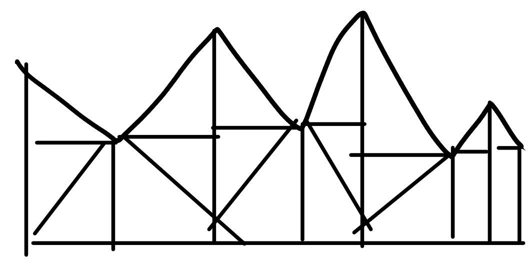
Seemingly unrelated : Area problem

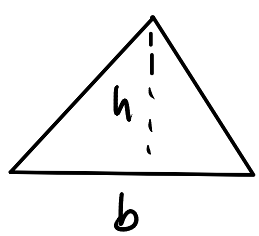


Find this area.



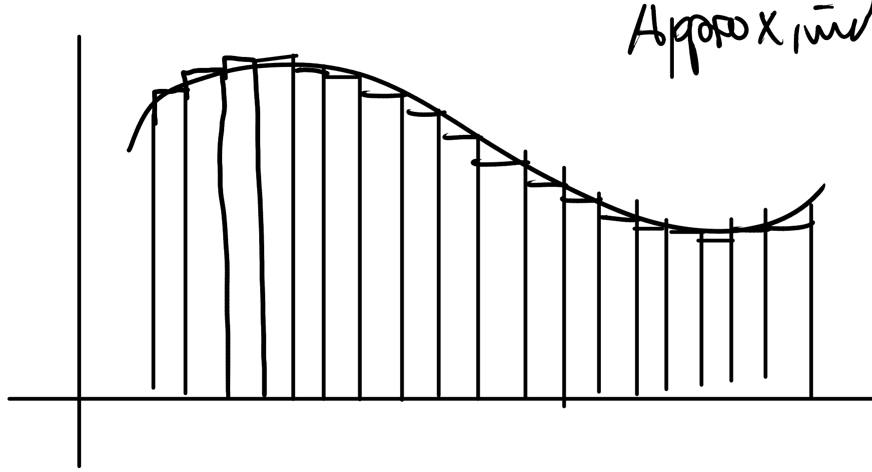
$A = WL$





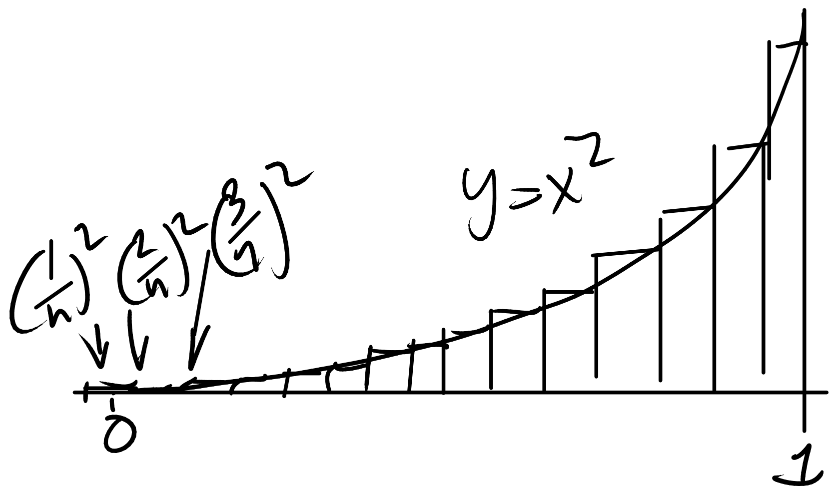
$A = \frac{1}{2}bh$

Approximate by rectangles



Area under graph $\approx$ Sum of the areas of the rectangles.

If we do the approximations sysmatically, and are somewhat clever, can determine the exact area using these approximations by rectangles.



n rectangles   width of each $= \frac{1}{n}$

height of k th one is $\left(\frac{k}{n}\right)^2$

Total $= \frac{1}{n}\left(\frac{1}{n}\right)^2 + \frac{1}{n}\left(\frac{2}{n}\right)^2 + \frac{1}{n}\left(\frac{3}{n}\right)^2 + \cdots + \frac{1}{n}\left(\frac{n}{n}\right)^2$