Exam 2 Next Thursday 10/31

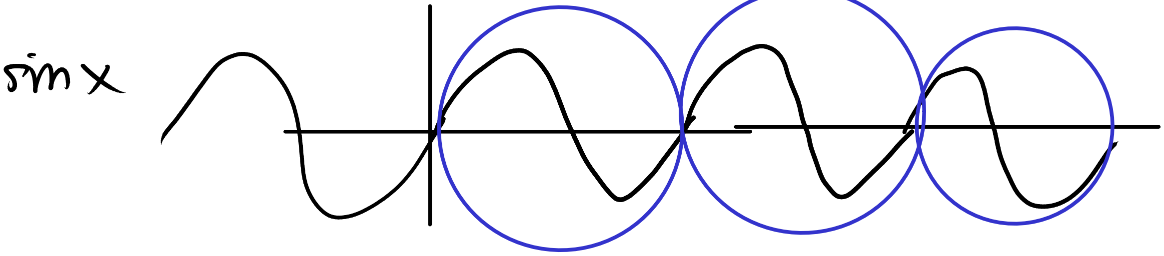
Homework Due 11/1 2:00pm

Curve Sketching: $f(x)$

- Domain
- Where is f zero, positive, negative?
- Symmetry even $f(-x) = f(x)$ eg. x^2 $\cos x$

  odd $f(-x) = -f(x)$ x^3 $\sin x$

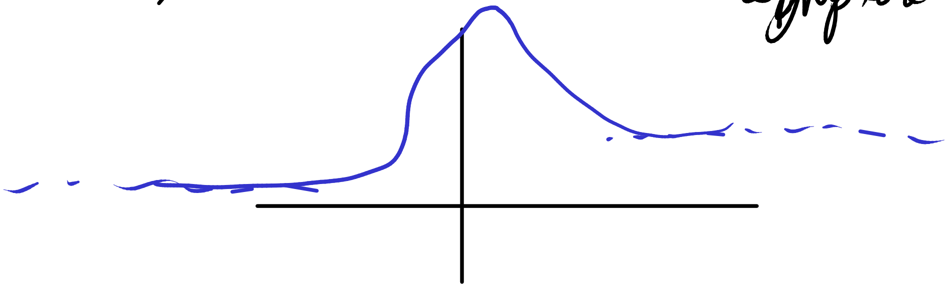
periodic function $f(x+c) = f(x)$



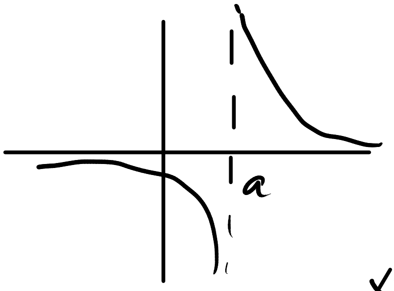
$\sin(x+2\pi) = \sin(x)$

Asymptotes $\lim_{x\to\infty} f(x) = L \Rightarrow$ horizontal asymptote at $y=L$

$\lim_{x\to-\infty} f(x)$



vertical asymptote



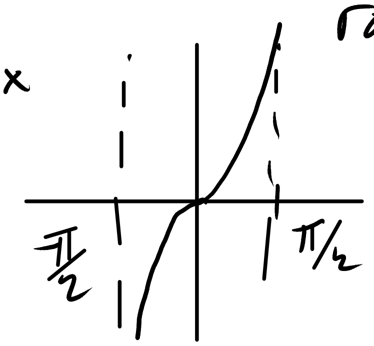
$\lim_{x \to a^+} f(x) = \infty$

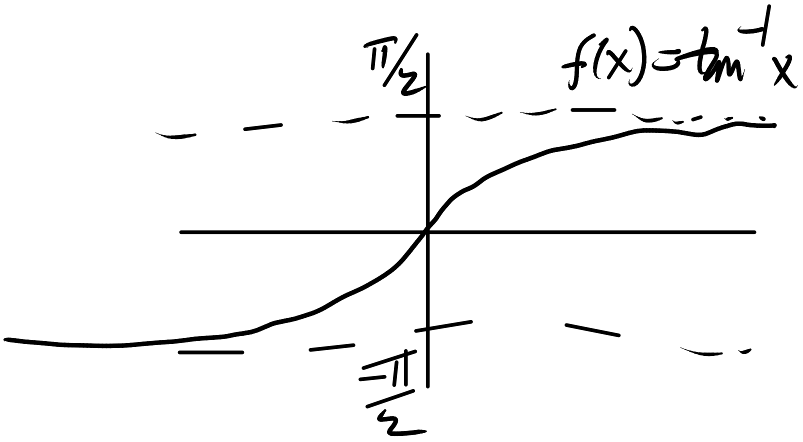
$\lim_{x \to a^-} f(x) = -\infty$

$f(x) = \tan^{-1} x$ domain $(-\infty, \infty)$

range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

tan x





f' where is it zero, positive or negative?
or undefined.

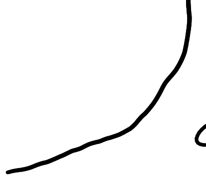
critical points = where f' is zero or undefined

f' is positive → f increasing
f' is negative → f decreasing

→ use this to find local maxima/minima

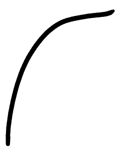
look at f''
where is f'' zero, positive or negative?

$f''(x) = 0$ inflection point.
$f''(x) > 0$ → f' is increasing →



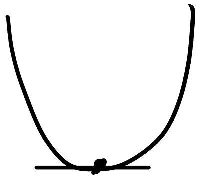
concave up

$f''(x) < 0 \rightarrow f'$ is decreasing $\rightarrow$



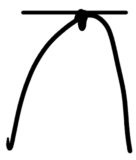
concave down

$f'(c) = 0$ and $f''(c) > 0$



There is a local minimum at c

$f'(c) = 0$ and $f''(c) < 0$



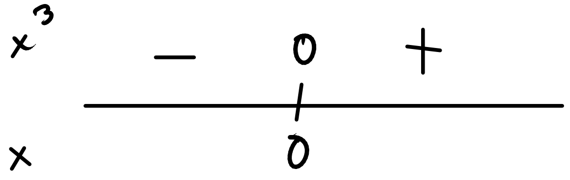
There is a local maximum at c.

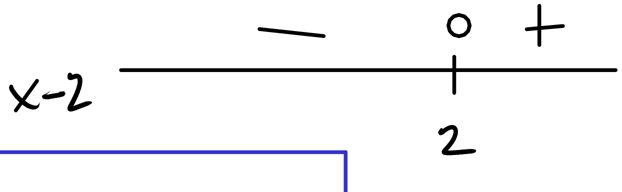
Graph $f(x) = \dfrac{x^3}{x-2}$

Domain all x except 2 $(-\infty,2) \cup (2,\infty)$

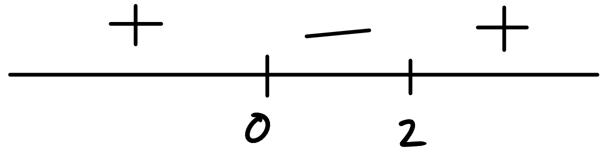
Where is f +, −, 0 ?

$f(0) = 0$





$\dfrac{x^3}{x-2}$



Asymptotes $\lim_{x\to\infty} \dfrac{x^3}{x-2}$ $\dfrac{\infty}{\infty}$ L'Hopital's $\lim_{x\to\infty} \dfrac{3x^2}{1} = \infty$

so $\lim_{x\to\infty} \dfrac{x^3}{x-2} = \infty$

$\lim_{x\to-\infty} \frac{x^3}{x-2}$ same argument $\lim_{x\to-\infty} \frac{3x^2}{1} = \infty$

$\boxed{\text{goes off to } +\infty \text{ at both ends}}$

Vertical asymptote at $x=2$

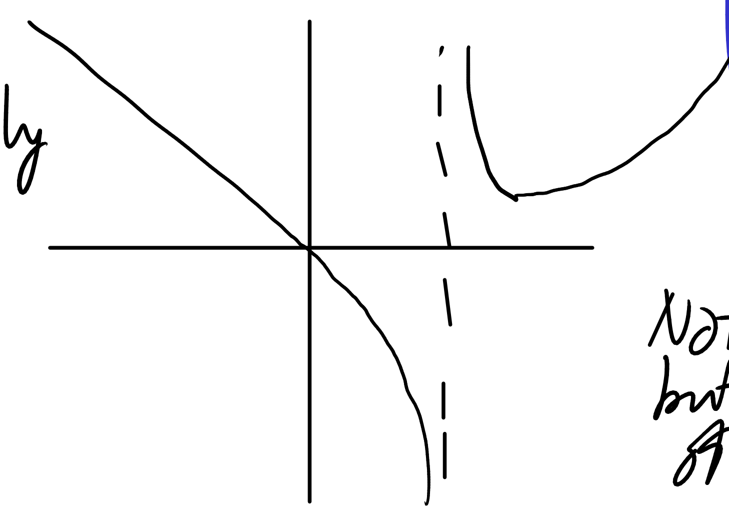
$\lim_{x\to 2^+} \frac{x^3}{x-2}$ $\frac{8}{0}$ $\Rightarrow$ infinite limit

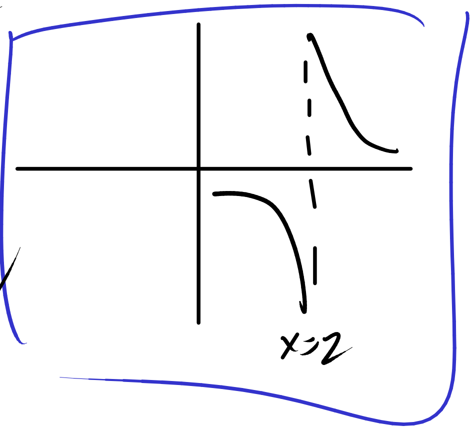
f is positive to the right of 2 so

$\lim_{x\to 2^+} \frac{x^3}{x-2} = +\infty$

$\lim_{x\to 2^-} \frac{x^3}{x-2} = -\infty$

Very roughly graph looks like.





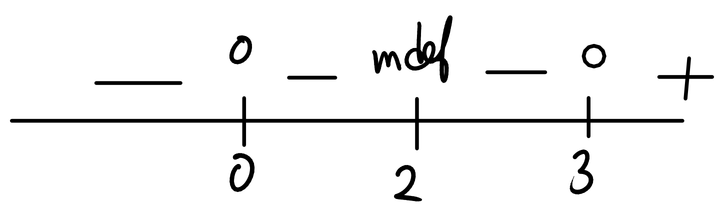
Not totally wrong but the graph has other features.

Derivative $f(x) = \frac{x^3}{x-2}$

$$f'(x) = \frac{(x-2)\,3x^2 - x^3(1)}{(x-2)^2} = \frac{3x^3-6x^2-x^3}{(x-2)^2}$$

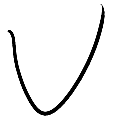
$$f'(x) = \frac{2x^3-6x^2}{(x-2)^2} = \frac{2x^2(x-3)}{(x-2)^2}$$

f'



local minimum at $x=3$ $f'(3)=0$

f' goes $-$ to $+$



$$f''(x) = \frac{d}{dx}\left(\frac{2x^2(x-3)}{(x-2)^2}\right)$$

$$= \frac{(x-2)^{\cancel{2}}(2)(2x(x-3)+x^2) - 2x^2(x-3)\,2\cancel{(x-2)}}{(x-2)^{\cancel{4}\,3}}$$

$$= \frac{(x-2)\,2\,(2x(x-3)+x^2) - 2x^2(x-3)\cdot 2}{(x-2)^3}$$

$$= \frac{(x-2)\cdot 2\cdot(2x^2-6x+x^2) - 4x^3+12x^2}{(x-2)^3}$$

$$= \frac{2(x-2)(3x^2-6x) - 4x^3+12x^2}{(x-2)^3}$$

$$= \frac{2x[\,(x-2)(3x-6)-2x^2+6x\,]}{(x-2)^3}$$

$$= \frac{2x[\,3x^2-6x-6x+12-2x^2+6x\,]}{(x-2)^3}$$

$$= \frac{2x[\,x^2-6x+12\,]}{(x-2)^3} = f''(x)$$

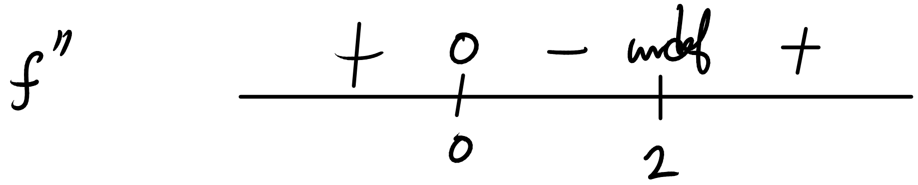
$f''(x)=0$ if $x=0$ or

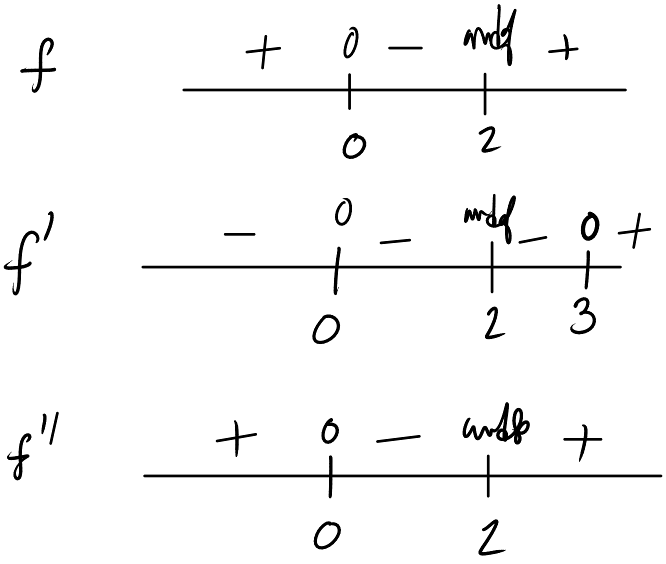
if $x^2-6x+12=0$ $\quad (-6)^2-4\cdot1\cdot12$

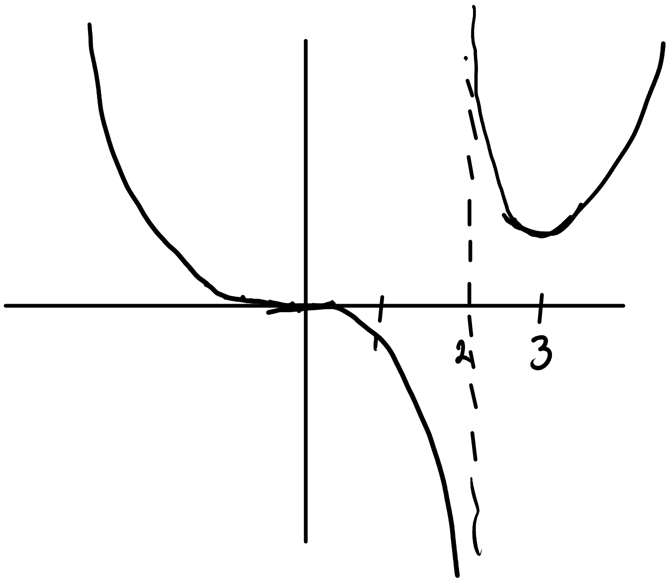
No solution $\quad 36-48<0$

$x^2-6x+12>0$

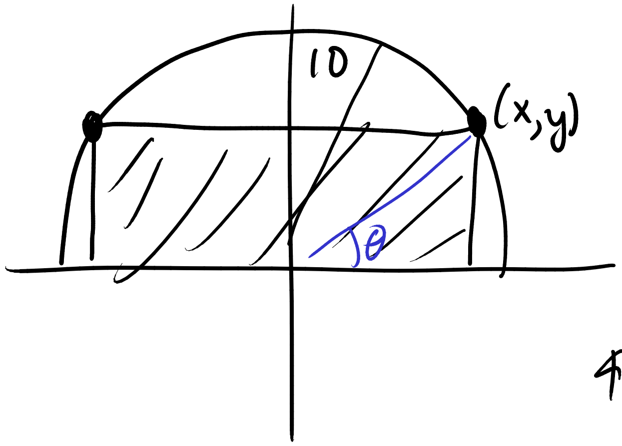
only inflection point is at zero?







Optimization = finding maxima and minima



find the largest Area of rectangle inscribed in a semicircle

introduce (x, y) that are coordinates of this corner point

height = y   width = 2x

$A = 2xy$

equation of semicircle

$x^2+y^2=10^2$ and $y \geq 0$

use equations to eliminate some variable

In this problem let's eliminate y

$y = \sqrt{10^2 - x^2}$ $A = 2x\sqrt{10^2 - x^2}$

$$0 = \frac{dA}{dx} = 2\sqrt{10^2-x^2} + \cancel{2}x\,\frac{1}{\cancel{2}}(10^2-x^2)^{-1/2}(-2x)$$

$$= 2\sqrt{10^2-x^2} - \frac{2x^2}{\sqrt{10^2-x^2}}$$

$$0 = \frac{2(10^2-x^2)-2x^2}{\sqrt{10^2-x^2}} = \frac{2\cdot 10^2-4x^2}{\sqrt{10^2-x^2}}$$

$$2\cdot 10^2-4x^2=0$$

$$10^2=2x^2$$

$$50=x^2$$

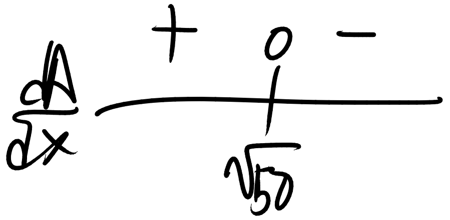
$$x=\sqrt{50}$$

$$A(x=\sqrt{50}) = 2\sqrt{50}\sqrt{100-50}$$

$$= 2\sqrt{50}\sqrt{50} = 100$$

$\frac{dA}{dx} < 0$ if $x > \sqrt{50}$

$\frac{dA}{dx} > 0$ if $x < \sqrt{50}$



So it really is a maximum