Marginal Cost

Marginal = Economist's derivative

Cost function cost to produce x units $\quad C(x) = 5 + 0.02x$

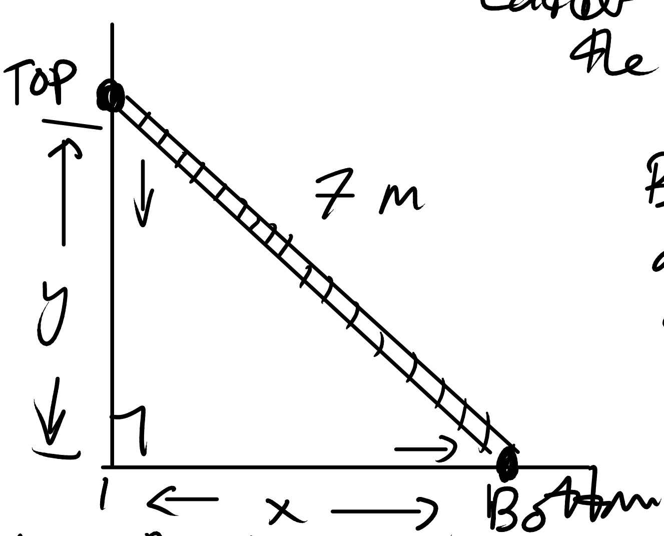
Marginal cost to produce 1 more unit

$$C'(x) = 0.02$$

$C(x) = x^2$ marginal cost to produce 1 more unit given that you have produced x units $C'(x) = 2x$

Related Rates.

Example:



Ladder is falling down the wall

Bottom is moving away from the wall at 1 m/s

How fast is the top moving when the bottom is 5 m from the wall.

$t =$ time (s)

$x =$ distance between bottom and wall

$\frac{dx}{dt} = 1$ m/s

$y =$ distance between floor and top.

want to know $\frac{dy}{dt}$. at the instant when $x = 5$

Pythagorean theorem $x^2 + y^2 = 7^2 = 49$

Implicit differentiation w.r.t. t

$$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(49) = 0$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2\cdot 5\cdot 1 + 2y\frac{dy}{dt} = 0$$

$$10 + 2\cdot 2\sqrt{6}\,\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{10}{2\cdot 2\sqrt{6}} = -\frac{5}{2\sqrt{6}}$$

$$x^2+y^2 = 7^2$$

$$5^2+y^2 = 7^2$$

$$25+y^2 = 49$$

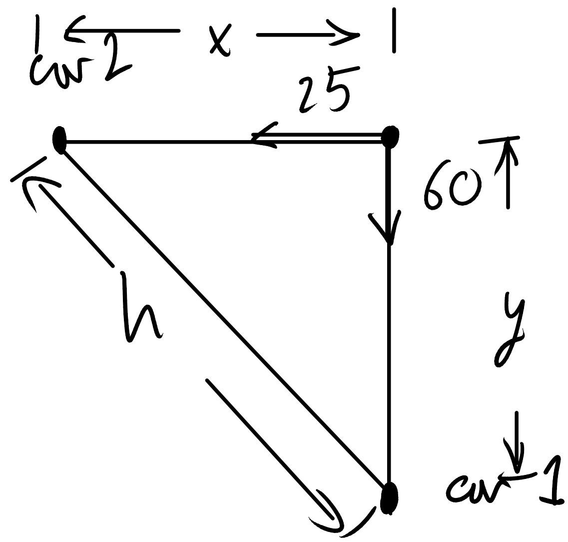
$$y^2 = 24 = 3\cdot 8 = 3\cdot 2^3$$

$$y = \sqrt{24} = 2\sqrt{6}$$

2) Two cars start at the same point.
One goes south at 60 mi/h
One goes west at 25 mi/h

At what rate is the distance between the cars increasing one hour later?

Step 1 draw a diagram



$$\frac{dx}{dt} = 25$$

$$\frac{dy}{dt} = 60$$

Want to know $\frac{dh}{dt}$

Find positions 1 hour later $\quad x = 25$, $y = 60$

relationship between x, y, h $\qquad h^2 = x^2 + y^2$

$$\frac{d}{dt}\left(h^2\right) = \frac{d}{dt}\left(x^2+y^2\right)$$

$$h^2 = 25^2 + 60^2$$
$$h = \sqrt{25^2+60^2} = 65$$

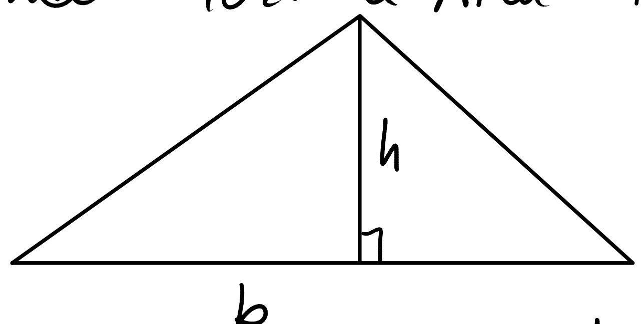
$$2h\frac{dh}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$2 \cdot 65 \frac{dh}{dt} = 2 \cdot 25 \cdot 25 + 2 \cdot 60 \cdot 60$$
$$2\left(65\frac{dh}{dt}\right) = 2\left(25^2+60^2\right)$$

$$\boxed{\frac{dh}{dt} = 65}$$

3) The altitude of a triangle is increasing at 1cm/min
area is increasing at 2 cm^2/min

At what rate is the base changing when
altitude = 10 cm & Area = 100 cm^2 ?



Area = A

$$A = \frac{1}{2} bh$$

$$\frac{dA}{dt} = \frac{d}{dt}\left(\frac{1}{2} bh\right) = \frac{1}{2}\left(\frac{db}{dt} h + b \frac{dh}{dt}\right)$$

$\frac{dh}{dt} = 1$ $\quad$ $\frac{dA}{dt} = 2$ $\quad$ $h = 10$

$A = 100$

$b = ?$ $\quad$ $100 = \frac{1}{2} \cdot b \cdot 10$

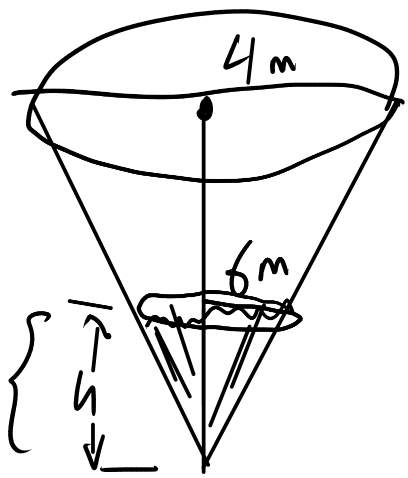
$100 = 5b$

$20 = b$

$$2 = \frac{1}{2}\left(\frac{db}{dt} \cdot 10 + 20 \cdot 1\right)$$

$4 = 10 \frac{db}{dt} + 20$ $\quad$ $10 \frac{db}{dt} = -16$ $\quad$ $\frac{db}{dt} = -1{,}6$

5) Water is leaking from a container at the same time as more water is being pumped in



Volume V

leak : 10000 cm^3/min

Suppose the water level is rising at 20 cm/min when the height is 2m.

How fast is water being pumped in?

Interested in the situation where $h = 2m$ and $\frac{dh}{dt} = .2\ m/min$

contribution from the leak

$$\frac{dV}{dt} = -10000 cm^3/min$$

$$= -.01\ m^3/min$$

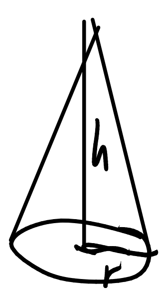
$100\ cm = 1\ m$

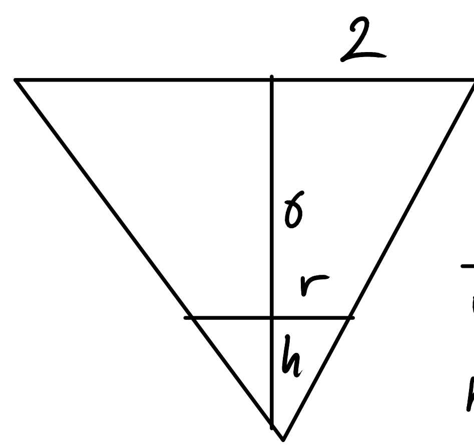
$(100)^3 cm^3 = 1\ m^3$

$\frac{1000000\ cm^3}{100} = \frac{1m^3}{100}$

$10000\ cm^3 = .01\ m^3$

$$V = \frac{1}{3}\pi r^2 h$$





$$\frac{h}{6} = \frac{r}{2}$$

$$r = \frac{1}{3}h$$

$$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$$

$$V = \frac{1}{27}\pi h^3$$

Water leaking and being pumped in

$$\frac{dV}{dt} = -0.01 \text{ m}^3/\text{min} + X$$

leak

rate at which water is pumped in.

$$V = \frac{1}{27}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{27}\pi \cdot 3h^2 \cdot \frac{dh}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt} = \frac{\pi}{9} 2^2 (.2)$$

$$\frac{dV}{dt} = \frac{\pi}{9} \cdot 4 \cdot \frac{2}{10} = \frac{8\pi}{90}$$

$$\frac{8\pi}{90} = -0.01 + X$$

$$X = \underbrace{\frac{8\pi}{90} + 0.01}_{\text{rate at which water is pumped in}} \text{ m}^3/\text{min}$$