More Derivative rules.

Exam I Next Tuesday Oct 1

Normal lecture room and time

Problems: mostly Quest + a few free response

Covers material including today

Last time product rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$

$$h(x) = \underbrace{(x^3+2x)}_{f(x)}\underbrace{e^x}_{g(x)} \qquad f'(x) = 3x^2+2$$

$$g'(x) = e^x$$

$$h'(x) = f(x)g'(x) + f'(x)g(x)$$

$$= (x^3+2x)e^x + (3x^2+2)e^x$$

$$= (x^3+3x^2+2x+2)e^x$$

$$h(x) = \frac{e^x}{\sqrt{x}} = e^x(x)^{-1/2}$$

$$h'(x) = \frac{d}{dx}\left[e^x (x)^{-1/2}\right] = \frac{d}{dx}[e^x]\cdot x^{-1/2} + e^x \cdot \frac{d}{dx}[x^{-1/2}]$$

$$= e^x \cdot (x)^{-1/2} + e^x \left(-\frac{1}{2}\right) x^{\left(-\frac{1}{2}-1\right)}$$

$$= e^x x^{-1/2} - \frac{1}{2} e^x x^{-3/2}$$

$$e^{2x} = (e^x)^2 = e^x \cdot e^x$$

$$\frac{d}{dx}\left(e^{2x}\right) = e^x \frac{d}{dx}[e^x] + \frac{d}{dx}[e^x]\, e^x$$

$$= (e^x)^2 + (e^x)^2 = 2e^{2x}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

low d-high less high d-low
draw a line, and below
the square of low must go.

$$y = \frac{x^3}{1-x^2} \qquad y' = \frac{(1-x^2)(3x^2) - (x^3)(-2x)}{(1-x^2)^2}$$

$h(x) = \dfrac{x^2}{f(x)}$ give $h'(x)$ in terms of $f(x)$ and $f'(x)$

$$h'(x) = \frac{f(x)(2x) - x^2 f'(x)}{f(x)^2} = \frac{2x}{f(x)} - x^2 \frac{f'(x)}{f(x)^2}$$

k constant

$$y = \frac{1}{x+ke^x} = \frac{(x+ke^x)\cdot 0 - 1\cdot(1+ke^x)}{(x+ke^x)^2}$$

$(x+he^x)' = 1+ke^x$

$$= \frac{-(1+ke^x)}{(x+ke^x)^2}$$

A derivation of quotient rule from product rule.

$$h(x) = \frac{f(x)}{g(x)} \qquad \text{Want } h'(x)$$

$$f(x) = h(x)\,g(x)$$

$\downarrow$ apply product rule

$$f'(x) = h'(x)\,g(x) + h(x)\,g'(x)$$

$$f'(x) = h'(x)\,g(x) + \frac{f(x)}{g(x)}\,g'(x)$$

solve for $h'(x)$

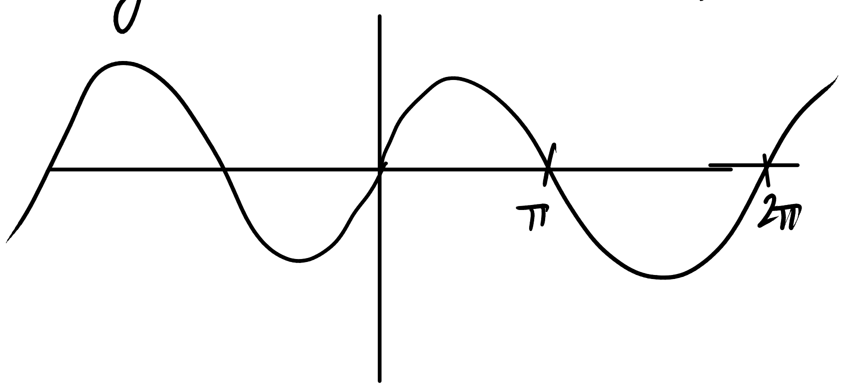
$$f'(x) - \frac{f(x)}{g(x)}\,g'(x) = h'(x)\,g(x)$$

$$\frac{1}{g(x)}\left[f'(x) - \frac{f(x)}{g(x)}\,g'(x)\right] = h'(x)$$

$$h'(x) = \frac{1}{g(x)}\left[\frac{g(x)f'(x)}{g(x)} - \frac{f(x)g'(x)}{g(x)}\right]$$

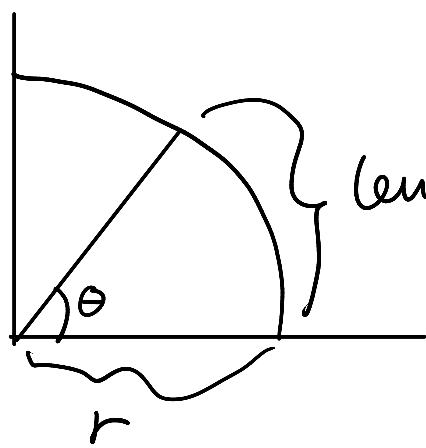
$$= \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Trig functions. $f(x) = \sin x$



Always use radians.

Radian



length $= r\theta$

True if θ is measured in radians.

~~degrees~~ ~~length of arc~~ $\cancel{= r\frac{\pi}{180}}$ ~~(# of degrees)~~

Derivative of $\sin x$ $\lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$$\sin(x+h) = \sin x \cos h + \cos x \sin h$$

$$\frac{\sin(x+h) - \sin(x)}{h} = \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

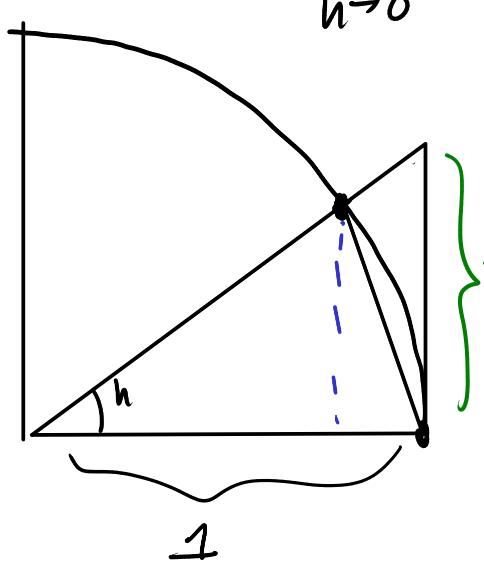
$$= \sin x \, \frac{\cos h - 1}{h} + \cos x \, \frac{\sin h}{h}$$

$$\lim_{h \to 0} \left(\sin x \, \frac{\cos h - 1}{h} + \cos x \, \frac{\sin h}{h} \right)$$

$$= \sin x \cdot \underbrace{\lim_{h \to 0} \frac{\cos h - 1}{h}}_{0} + \cos x \cdot \underbrace{\lim_{h \to 0} \frac{\sin h}{h}}_{1}$$

$$= \cos x \qquad \frac{d}{dx}[\sin x] = \cos x$$

Famous limit $\displaystyle \lim_{h \to 0} \frac{\sin h}{h} = 1$



Area of big triangle $= \frac{1}{2} \tan h$

Area of pie wedge $= \pi \frac{h}{2\pi}$

$= \frac{h}{2}$

Area of small triangle $= \frac{1}{2} \sin h$

$$\frac{1}{2}\sin h < \frac{h}{2} < \frac{1}{2}\tan h$$

$$\sin h < h < \tan h = \frac{\sin h}{\cos h}$$

$$\frac{\sin h}{h} < 1 < \frac{\sin h}{h}\frac{1}{\cos h}$$

$$\left.\begin{aligned} \cos h < \frac{\sin h}{h} < 1 \\ \lim_{h\to 0} \cos h = \cos 0 = 1 \end{aligned}\right\} \Rightarrow \text{apply squeeze theorem.} \quad \lim_{h\to 0}\frac{\sin h}{h} = 1$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos x\,(\sin x)' - \sin x(\cos x)'}{(\cos x)^2}$$

$$= \frac{\cos x \cos x - \sin x\,(-\sin x)}{(\cos x)^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x(-\sin x) - \cos x \cos x}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{\cos x \cdot 0 - 1\cdot(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}\,\frac{\sin x}{\cos x} = \sec x \cdot \tan x$$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{\sin x \; 0 - 1(\cos x)}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin^2 x} = -\frac{1}{\sin x}\,\frac{\cos x}{\sin x} = -\csc x \cot x$$

$$\frac{d}{dx}\left[3x^2 - 2\cos x\right] = 6x - 2(-\sin x)$$

$$= 6x + 2\sin x$$

$$\frac{d}{dx}\left[\sqrt{x}\,\sin x\right] = \sin x\,\frac{d}{dx}[\sqrt{x}] + \sqrt{x}\,\frac{d}{dx}[\sin x]$$

$$= \sin x\left(\frac{1}{2}x^{-1/2}\right) + \sqrt{x}\cos x$$

$$= \frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x$$

$$\frac{d}{dx}\left[\frac{\cot x}{e^x}\right] = \frac{e^x \frac{d}{dx}[\cot x] - \cot x \frac{d}{dx}[e^x]}{(e^x)^2}$$

$$= \frac{e^x(-\csc^2 x) - \cot x \cdot e^x}{(e^x)^2}$$

$$= \frac{-\csc^2 x - \cot x}{e^x}$$

<u>chain rule:</u> $\quad y = f(u) \qquad u = g(x)$

$$y = f(g(x)) = (f \circ g)(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))\, g'(x)$$

$y = \sin(x^2) \qquad u = x^2$

$\qquad\qquad\qquad\quad y = \sin u$

$\frac{dy}{du} = \cos u = \cos x^2 \qquad \frac{dy}{dx} = \cos(x^2) \cdot 2x$

$\frac{du}{dx} = 2x$