HW problem discussion.

Know $g(g(x)) = g(x)$ ←

$\underbrace{2}\ g$'s $= 1\ g$

Suppose we know $\underbrace{g\circ g\circ g\cdots\circ g}_{k} = \underset{1}{g}$ $\Rightarrow$

Prove $\underbrace{g\circ g\cdots\circ g}_{k+1} = g\circ(\underbrace{g\circ\cdots\circ g}_{k}) = g\circ g = g$

If $g(x) = x$ $\quad g(g(x)) = g(x)$

$g(x) = -x+1 \qquad g(0) = 1 \qquad g(1) = 0$

$g(g(x)) = -(-x+1)+1 = x \neq g(x)$

Why No? Suppose $g(0)=1$ $g(1)=0$ $\boxed{g(g(x))=g(x)}$

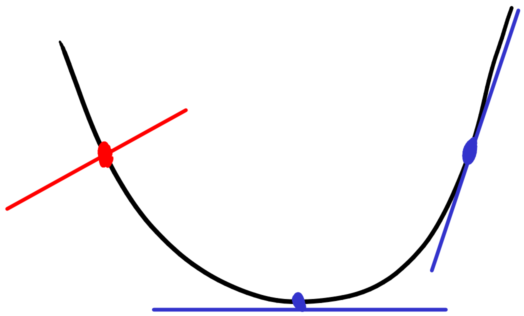
evaluate $g(0)$ evaluate $g(1)$

$g(g(0)) \boxed{=} g(1) \boxed{=} 0$

$\boxed{\parallel}$

property of g $\quad g(0) = 1 \qquad 0=1$ contradiction.

Tangents and velocities



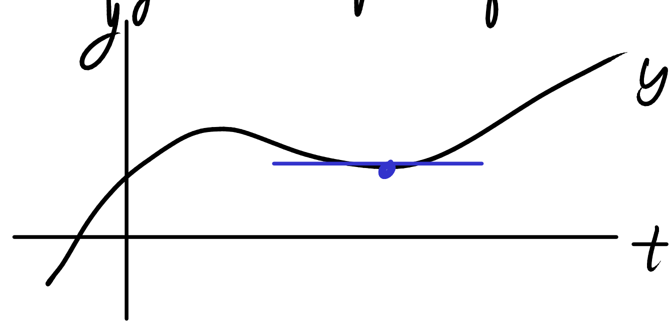
Q: How to compute tangents.

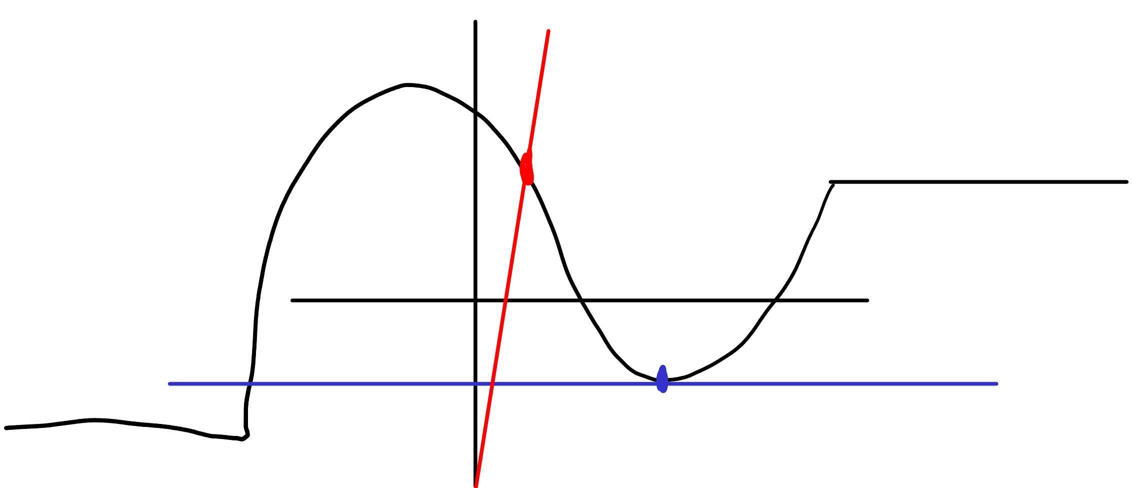
Particle moving in one dimension y

$y=f(t)$ is position as a function of t

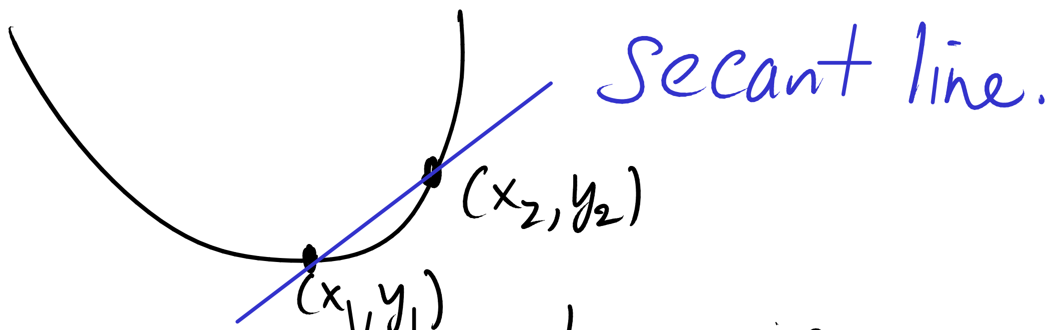
Q: <u>Instantaneous velocity</u> of the particle (speedometer reading)

Relationship: if you graph $y=f(t)$
velocity = slope of the tangent line to the graph





Need two points to determine a line



$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Average velocity between (t_1, y_1) (t_2, y_2)

$$\text{average vel} = \frac{y_2 - y_1}{t_2 - t_1}$$

e.g. $y(t) = 2t^2$ average velocity from $t=0$ to $t=1$

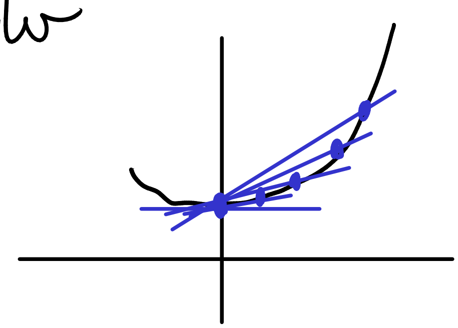
$t=0$ $y=0$

$t=1$ $y=2$

$$\text{average vel} = \frac{2-0}{1-0} = 2$$

instantaneous velocity $\approx$ average velocity when t_1 and t_2 are very close together.

Tangent: take secant line for two points very close together

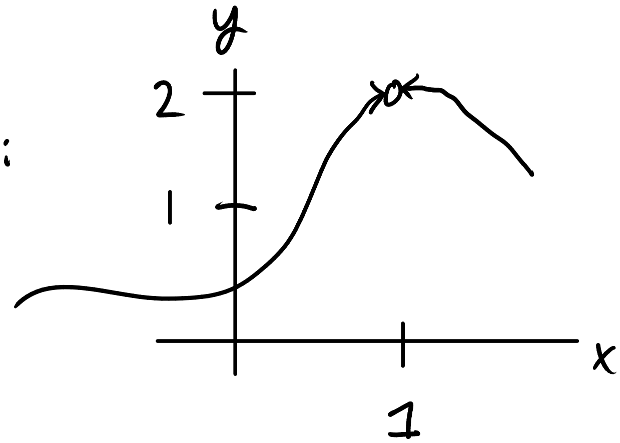


slope of tangent line = $\underline{\text{limit}}$ of slopes of secant lines

Limit of a function

Representative Picture :

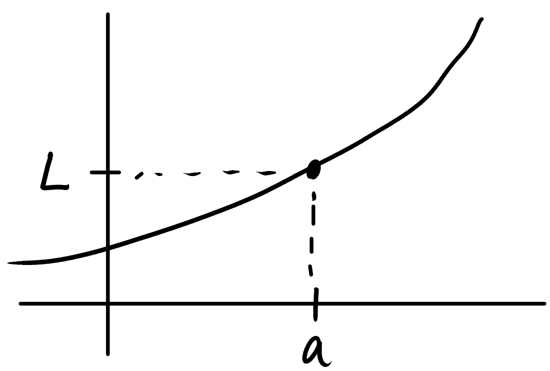
$f(1)$ is undefined



" values of $f(x)$ near $x=1$ are close to $y=2$ "

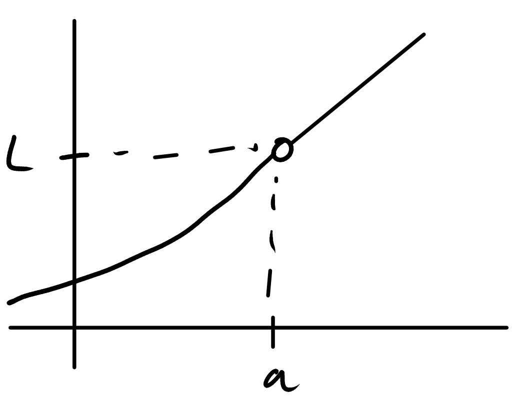
" as x approaches 1, $f(x)$ approaches 2 "

Symbolically $\lim_{x \to 1} f(x) = 2$



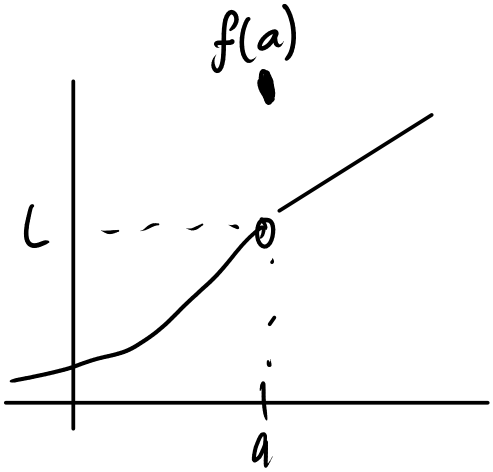
$\lim_{x \to a} f(x) = L$

and $f(a) = L$



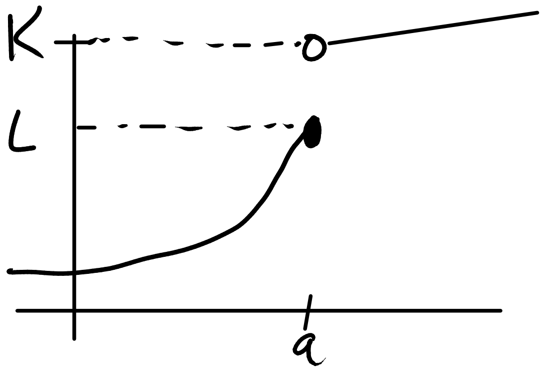
$\lim_{x \to a} f(x) = L$

and $f(a)$ is undefined



$$\lim_{x\to a} f(x) = L$$

and $f(a)$ is defined $f(a) \neq L$ and we don't care.



$\lim_{x\to a} f(x)$ does not exist.

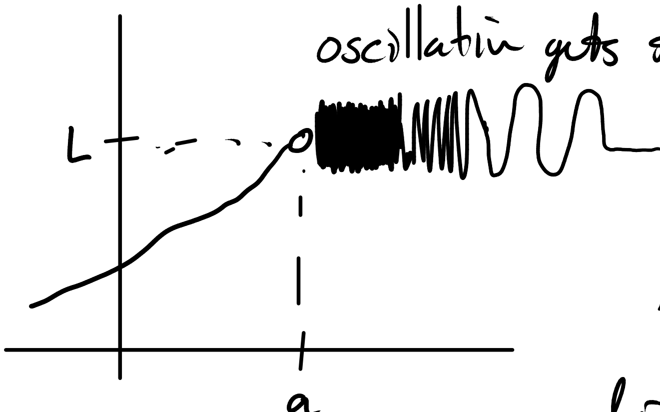
limit from the left

$$\lim_{x\to a^-} f(x) = L$$

limit from the right

$$\lim_{x\to a^+} f(x) = K$$

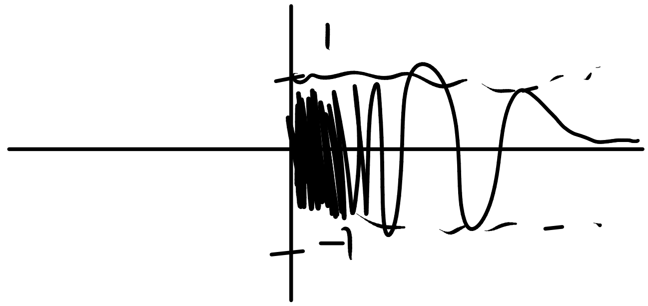
oscillation gets faster and faster.

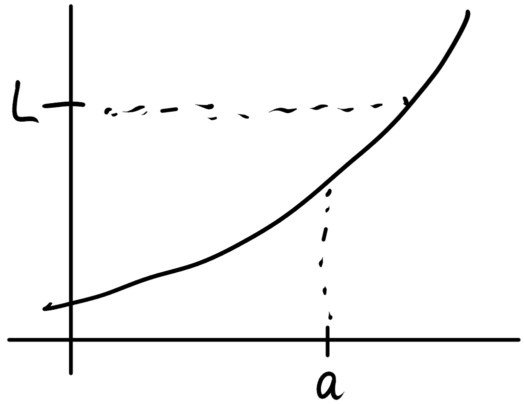


$$\lim_{x\to a^-} f(x) = L$$

$\lim_{x\to a^+} f(x)$ does not exist.

$f(x) = \sin \frac{1}{x}$





$\lim_{x \to a} f(x) \neq L$

Limit rules: Suppose c is a constant and assume that $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist (and are finite numbers)

then

1) $\lim_{x \to a} [f(x) + g(x)] = [\lim_{x \to a} f(x)] + [\lim_{x \to a} g(x)]$

2) $\lim_{x \to a} [c f(x)] = c [\lim_{x \to a} f(x)]$

3) $\lim_{x \to a} [f(x) \cdot g(x)] = [\lim_{x \to a} f(x)] \cdot [\lim_{x \to a} g(x)]$

4) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ provided that is not zero.

Theorem $\lim_{x \to a} x = a$

$$\lim_{x \to a} x^2 \overset{\text{by (3)}}{=} \left(\lim_{x \to a} x\right) \cdot \left(\lim_{x \to a} x\right) = a \cdot a = a^2$$

$$\lim_{x \to a} x^n \overset{\text{by (3)}}{=} \left(\lim_{x \to a} x\right)^n = a^n$$

$$\lim_{x \to a} cx^n \overset{\text{by (2)}}{=} ca^n$$

$$\lim_{x \to a} c_n x^n + c_{n-1} x^{n-1} + \cdots + c_1 x + c = c_n a^n + c_{n-1} a^{n-1} + \cdots + c_1 a + c_0$$

c_i are constants