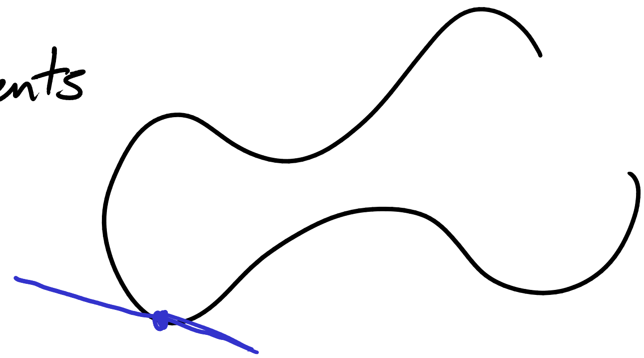
408C Calculus 17th century

Galileo & Kepler

d. 1642 d. 1630

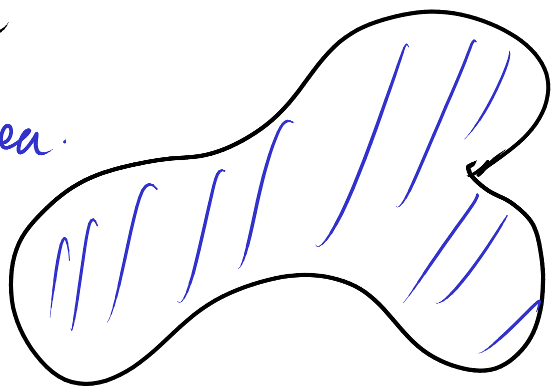
Two problems : Tangents



Q: Find the tangent line

Quadrature

Find the area.



Newton + Leibniz
These two problems are closely related!

Newton discovery 1666 published 1687

Leibniz discovery 1675 published 1684

$\int f\,dx$ Leibniz notation.

Function: correspondence between two sets

Function f, correspondence $y = f(x)$

x = independent variable / input

y = dependent variable / output

$f(\ )$ itself is the correspondence by which the input is sent to the output.

Ex $f(x) = x^2+2x-3$ formula tells you how to compute the output from the input.

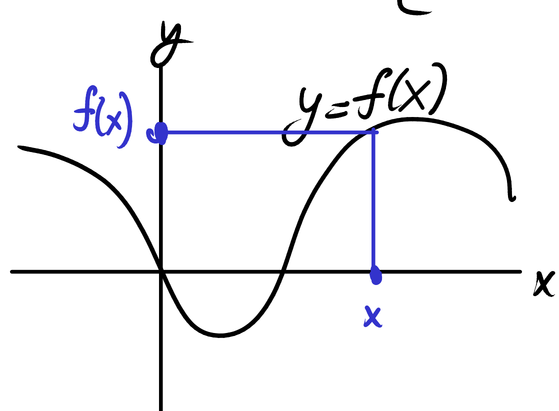
program: $g(x) = \begin{cases} \text{if } x=0 \\ \quad \text{return } 1. \\ \text{else} \\ \quad \text{return } x \cdot g(x-1) \end{cases}$

if $n \geq 0$ is an integer $g(n) = n!$

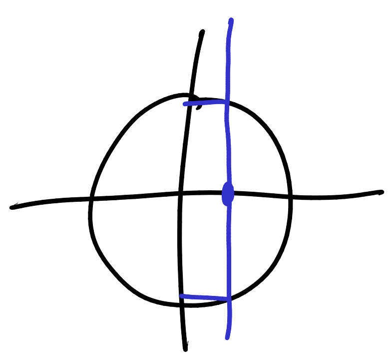
$$n! = n \cdot (n-1)!$$

$$n(n-1)(n-2)\cdots 3\cdot 2\cdot 1 = n\left[(n-1)(n-2)\cdots 3\cdot 2\cdot 1\right]$$

graph:



plot all points $(x, f(x))$



Not a function because vertical line intersects graph twice.

Vertical line test: graph of a function intersects each vertical line at most one time.

Table:

n	$g(n)$
0	1
1	1
2	2
3	6
4	24
5	120

Numbers: Integers = $0, 1, 2, 3, \ldots$
$-1, -2, -3, \ldots$

Rational number = $\frac{n}{m}$ $\quad \frac{1}{2}, \frac{2}{3}, \frac{-5}{11}, \frac{4}{1} = 4$

Real number = decimals $0.123592174\ldots$
(can define precisely) $e = 2.718281828\ldots$
$\pi = 3.14159\ldots$

$\mathbb{R}$ = all real numbers = $(-\infty, \infty)$

$(a,b) = \{x$ such that $a < x < b\}$

$[a,b] = \{x$ such that $a \le x \le b\}$

$[a,b) = \{ a \le x < b \}$ a [——) b

$(a,b] = \{ a < x \le b \}$ a (——] b

$(-\infty,a) = \{ x < a \}$

$(-\infty,a] = \{ x \le a \}$

$(a,\infty) = \{ a < x \}$

$[a,\infty) = \{ a \le x \}$

$(-\infty,\infty) = \{$ no condition on $x \}$

Domain of a function = possible values of x.

Range of a function = possible values of y

$y = f(x) = x^2$ Domain $= (-\infty,\infty)$
Range $= [0,\infty)$

$y = f(x) = \frac{1}{x}$ $x=0$ is forbidden
Domain $= (-\infty,0) \cup (0,\infty)$

$\cup$ denotes union. (OR)

Range $= (-\infty,0) \cup (0,\infty)$

$y = f(x) = \sqrt{1-x}$ Domain

Can't take $\sqrt{\ }$ of negative: need $1 - x \geq 0$

$1 \geq x$

$x \leq 1$

$x \in (-\infty, 1]$

[S abstract set

$x \in S$ means that x is an element of S]

$x \in (a,b)$ means x is a number in the interval (a,b).

Domain $= (-\infty, 1]$ Range $= [0, \infty)$

prime counting function $\pi(n)$

$\pi(n)$ = the number of primes $\leq n$

primes $= \{2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots$

$\pi(2) = 1$ $\pi(3) = 2$ $\pi(20) = 8$

PNT if n is large $\pi(n) \approx \dfrac{n}{\ln n}$

Exponential function:

Fix a number $a > 0$ Want to define a^x for a real number x.

n is an integer (positive) $a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ times}}$.

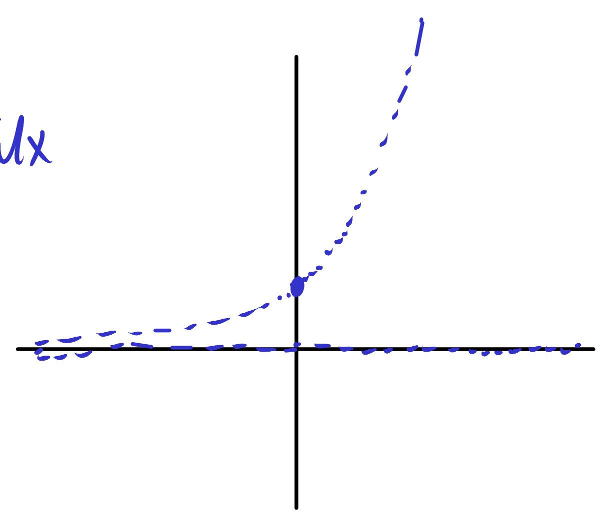
$a^{-1} = \frac{1}{a}$ $a^{-n} = \frac{1}{a^n} = \frac{1}{\underbrace{a \cdot a \cdots a}_{n \text{ times}}}$

$a^{1/n} = \sqrt[n]{a}$

$\frac{n}{m}$ is rational $a^{n/m} = \left(a^{1/m}\right)^n = \left(\sqrt[m]{a}\right)^n$

Assume $a > 1$

plot a^x for rational x



Calculus proves it is possible to fill this in to a solid line (limit, continuous function)

Stewart §1.5   1-4, 11-16, 19-20   ungraded HW