M 408C Exam 2 Free Response (version A)
October 31, 2013
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NAME:

EID:

INSTRUCTIONS:

- No books, notes, calculators, or other electronic devices. Do not look at anyone else's paper during the test.
- You must show your work to receive full credit. No credit will be awarded for a mere numerical answer, even if the number is correct.
- Only answers written on these sheets will be graded. Work written on scratch paper will not be considered.

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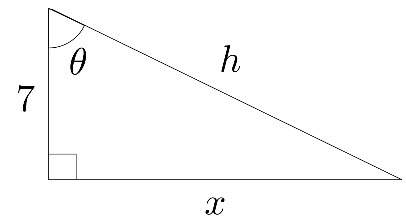
Problem	Possible	Actual
1	25	
2	25	
Total	50	

1. (25 points) A triangle is shown. One of the sides has a fixed length, while the sides labeled x and h are allowed to vary, and the angle θ varies as well.



Show that

$$\frac{dh}{dx} = \sin\theta$$

Pythagorean theorem $\quad 7^2 + x^2 = h^2$

Implicit diff. $\quad \frac{d}{dx}\left(7^2 + x^2\right) = \frac{d}{dx}\left(h^2\right)$

$$2x = 2h\frac{dh}{dx}$$

$$\Longrightarrow \quad \frac{dh}{dx} = \frac{2x}{2h} = \frac{x}{h} = \frac{\text{opp.}}{\text{hyp.}} = \sin\theta$$

2. (25 points) Find the absolute minimum value of the function

$$f(x) = x + \frac{4}{x}$$

defined on the domain $0 < x < \infty$, and find the value(s) of x where that minimum value is achieved. In order to receive full credit, you must not only find the local minimum, but also explain why this local minimum is in fact an absolute minimum.

Critical points: $f'(x) = 1 - \frac{4}{x^2}$

$$1 - \frac{4}{x^2} = 0 \iff 1 = \frac{4}{x^2} \iff x^2 = 4 \iff x = \pm 2$$

only $x=2$ is in the domain $0 < x < \infty$

Sign of $f'(x)$: $-$ on $(0, 2)$, $+$ on $(2, \infty)$

Test points: $f'(1) = 1 - \frac{4}{1} = -3 < 0$ $\quad f'(3) = 1 - \frac{4}{9} = \frac{5}{9} > 0$

Since $f'(x)$ goes from $-$ to $+$ at $x=2$, this is a local minimum

[Alternative: $f''(x) = \frac{8}{x^3}$, and $f''(2) = 1 > 0 \Rightarrow$ local min]

Furthermore, since $f'(x) > 0$ to the right of $x=2$ $f(x)$ is increasing to the right of $x=2$, an never become less than $f(2)$

Since $f'(x) < 0$ to the left of $x=2$, $f(x)$ is decreasing there, and it is always greater than $f(2)$.

Therefore, $f(2) = 4$ is the absolute minimum.