M 408C Exam 1 Free Response (version B)
October 1, 2013
Instructor: James Pascaleff

NAME: SOLUTIONS

EID:

INSTRUCTIONS:

- No books, notes, calculators, or other electronic devices. Do not look at anyone else's paper during the test.
- You must show your work to recieve full credit. No credit will be awarded for a mere numerical answer, even if the number is correct.
- Only answers written on these sheets will be graded. Work written on scratch paper will not be considered.

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Problem	Possible	Actual
1	25	
2	25	
Total	50	

1. (25 points) Using the properties of limits that we have discussed in class, determine the limit

$$\lim_{x \to 0} \frac{e^{2x} - 3e^x + 2}{e^{2x} - 1}$$

To recieve full credit, you must show the steps that you use to arrive at your answer.
Warning: No credit will be awarded for a solution using L'Hopital's rule.
Hint: Let $z = e^x$. Then $e^{2x} = z^2$. Try to simplify.

$$\frac{e^{2x} - 3e^x + 2}{e^{2x} - 1} = \frac{z^2 - 3z + 2}{z^2 - 1} = \frac{(z-1)(z-2)}{(z-1)(z+1)}$$

$$= \frac{z-2}{z+1} = \frac{e^x - 2}{e^x + 1}$$

$$\lim_{x \to 0} \frac{e^{2x} - 3e^x + 2}{e^{2x} - 1} = \lim_{x \to 0} \frac{e^x - 2}{e^x + 1} = \frac{e^0 - 2}{e^0 + 1} = \frac{1-2}{1+1} = \frac{-1}{2}$$

Could also say, as $x \to 0$, $z = e^x \to e^0 = 1$

thus $\lim_{x \to 0} \frac{e^{2x} - 3e^x + 2}{e^{2x} - 1} = \lim_{z \to 1} \frac{z^2 - 3z + 2}{z^2 - 1}$

$$= \lim_{z \to 1} \frac{(z-1)(z-2)}{(z-1)(z+1)} = \lim_{z \to 1} \frac{z-2}{z+1} = \frac{1-2}{1+1} = \frac{-1}{2}$$

2. (25 points total)

(a) (12 points) Consider three functions $f(x)$, $g(x)$ and $h(x)$. By combining the product rule and the quotient rule, find a rule for the derivative of $F = fg/h$. That is, express

$$F' = \left(\frac{fg}{h}\right)'$$

in terms of f, g, h, f', g', and h'. Show the steps in your argument.

$$F' = \left(\frac{fg}{h}\right)' = \frac{h(fg)' - fgh'}{h^2}$$ Quotient rule

$$= \frac{h(f'g + fg') - fgh'}{h^2}$$ product rule applied to $(fg)'$

Alternative: let $u = fg$ then $F = \frac{u}{h}$

$$F' = \left(\frac{u}{h}\right)' = \frac{hu' - uh'}{h^2}$$ Quotient rule

And $u' = f'g + fg'$ by product rule

So $F' = \dfrac{h(f'g + fg') - fgh'}{h^2}$

(b) (13 points) Use this formula to compute $F'(0)$ in the situation where f, g and h are the specific functions

$$\begin{aligned} f(x) &= \sin x + \cos x \\ g(x) &= (1 + x + 2x^2) \\ h(x) &= e^x \end{aligned}$$

Please note that you *are not required* to compute $F'(x)$ as a function of x, you just need to compute $F'(0)$.

$f(0) = 1 \qquad f'(x) = \cos x - \sin x \qquad f'(0) = 1$

$g(0) = 1 \qquad g'(x) = 1 + 4x \qquad g'(0) = 1$

$h(0) = 1 \qquad h'(x) = e^x \qquad h'(0) = 1$

$$F'(0) = \frac{h(0)\left(f'(0)g(0) + f(0)g'(0)\right) - f(0)g(0)h'(0)}{(h(0))^2}$$

$$= \frac{1\left[1\cdot 1 + 1\cdot 1\right] - 1\cdot 1\cdot 1}{1^2}$$

$$= \frac{[1+1] - 1}{1} = 1$$