**M 408C Exam 1 Free Response (version A)**
**October 1, 2013**
**Instructor: James Pascaleff**

**NAME:** SOLUTIONS

**EID:**

**INSTRUCTIONS:**

- No books, notes, calculators, or other electronic devices. Do not look at anyone else's paper during the test.
- You must show your work to recieve full credit. No credit will be awarded for a mere numerical answer, even if the number is correct.
- Only answers written on these sheets will be graded. Work written on scratch paper will not be considered.

FOR OFFICIAL USE ONLY:

| Problem | Possible | Actual |
|---|---|---|
| 1 | 25 | |
| 2 | 25 | |
| Total | 50 | |

1. (25 points) Using the properties of limits that we have discussed in class, determine the limit

$$\lim_{x \to 0} \frac{e^{2x} + e^x - 2}{e^{2x} - 1}$$

To recieve full credit, you must show the steps that you use to arrive at your answer.
*Warning:* No credit will be awarded for a solution using L'Hopital's rule.
*Hint:* Let $z = e^x$. Then $e^{2x} = z^2$. Try to simplify.

$$\frac{e^{2x} + e^x - 2}{e^{2x} - 1} = \frac{z^2 + z - 2}{z^2 - 1} = \frac{(z-1)(z+2)}{(z-1)(z+1)}$$

$$= \frac{z+2}{z+1} = \frac{e^x + 2}{e^x + 1} \quad (\text{as long as } x \neq 0)$$

$$\lim_{x \to 0} \frac{e^{2x} + e^x - 2}{e^{2x} - 1} = \lim_{x \to 0} \frac{e^x + 2}{e^x + 1} = \frac{e^0 + 2}{e^0 + 1} = \frac{1+2}{1+1} = \frac{3}{2}$$

Could also say, as $x \to 0$, $z = e^x \to e^0 = 1$

thus $\lim_{x \to 0} \frac{e^{2x} + e^x - 2}{e^{2x} - 1} = \lim_{z \to 1} \frac{z^2 + z - 2}{z^2 - 1}$

$$= \lim_{z \to 1} \frac{(z-1)(z+2)}{(z-1)(z+1)} = \lim_{z \to 1} \frac{z+2}{z+1} = \frac{1+2}{1+1} = \frac{3}{2}$$

2. (25 points total)

   (a) (12 points) Consider three functions $f(x)$, $g(x)$ and $h(x)$. By combining the product rule and the quotient rule, find a rule for the derivative of $F = f/(gh)$. That is, express

$$F' = \left(\frac{f}{gh}\right)'$$

in terms of $f, g, h, f', g'$, and $h'$. Show the steps in your argument.

$$F' = \left(\frac{f}{gh}\right)' = \frac{ghf' - f(gh)'}{(gh)^2}$$ Quotient rule

$$= \frac{ghf' - f(g'h + gh')}{(gh)^2}$$ product rule applied to $(gh)'$

Alternative: let $u = gh$ then $F = \frac{f}{u}$

$$F' = \left(\frac{f}{u}\right)' = \frac{uf' - fu'}{u^2}$$ Quotient rule

And, $u' = (gh)' = g'h + gh'$ by product rule

So $$F' = \frac{ghf' - f(g'h + gh')}{(gh)^2}$$

(b) (13 points) Use this formula to compute $F'(0)$ in the situation where $f$, $g$ and $h$ are the specific functions

$$f(x) = \sin x + \cos x$$
$$g(x) = e^x$$
$$h(x) = (1 + x^2)$$

Please note that you *are not required* to compute $F'(x)$ as a function of $x$, you just need to compute $F'(0)$.

$f(0) = 1$ $\quad$ $f'(x) = \cos x - \sin x$ $\quad$ $f'(0) = 1$

$g(0) = 1$ $\quad$ $g'(x) = e^x$ $\quad$ $g'(0) = 1$

$h(0) = 1$ $\quad$ $h'(x) = 2x$ $\quad$ $h'(0) = 0$

$$F'(0) = \frac{g(0)h(0)f'(0) - f(0)[g'(0)h(0) + g(0)h'(0)]}{[g(0)h(0)]^2}$$

$$= \frac{1 \cdot 1 \cdot 1 - 1 \cdot [1 \cdot 1 + 1 \cdot 0]}{(1 \cdot 1)^2}$$

$$= \frac{1 - (1 + 0)}{1} = \frac{1-1}{1} = 0$$