

- Show all work. No books, notes, calculators, or other electronic devices.

(10 points) Find the eigenvalues and eigenvectors of the matrix:

$$A = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \quad (1)$$

$$\det(A - rI) = \det \begin{pmatrix} 5-r & -1 \\ 3 & 1-r \end{pmatrix} = (5-r)(1-r) + 3$$

$$= 5 - 6r + r^2 + 3 = r^2 - 6r + 8 = (r-2)(r-4)$$

so eigenvalues are $r_1 = 2$ and $r_2 = 4$

For $r_1 = 2$ eigenvector $\begin{pmatrix} 3 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 0$

so $v_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ is an eigenvector for $r_1 = 2$

For $r_2 = 4$, $\begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$

so $v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector for $r_2 = 4$