NAME & EID: Solutions

M 427K Quiz 8 **October 31, 2012** **Instructor: James Pascaleff**

- Show all work. No books, notes, calculators, or other electronic devices.

1. (5 points) Find the inverse Laplace transform of

$$F(s) = \frac{2s-3}{s^2-4} \tag{1}$$

Partial fractions: $s^2 - 4 = (s-2)(s+2)$

$\frac{2s-3}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2} \rightsquigarrow 2s-3 = A(s+2) + B(s-2)$

plug in $s=-2$: $-4-3 = B(-4)$ $B = 7/4$

plug in $s=2$: $4-3 = A(4)$ $A = 1/4$

$f(t) = Ae^{2t} + Be^{-2t} = \frac{1}{4}e^{2t} + \frac{7}{4}e^{-2t}$ OR $F(s) = 2\left(\frac{s}{s^2-4}\right) - \frac{3}{2}\left(\frac{2}{s^2-4}\right)$

$f(t) = 2\cosh(2t) - \frac{3}{2}\sinh(2t)$

2. (5 points) Consider the initial value problem

$$y'' - 2y' + 2y = \cos t, \quad y(0) = 1, \quad y'(0) = 0. \tag{2}$$

Solve for the Laplace transform $Y(s) = \mathcal{L}\{y(t)\}$ of the solution to the problem. Your answer should be a function of s. You do not need to take the inverse transform.

$\mathcal{L}\{y'\} = sY(s) - y(0) = sY(s) - 1$

$\mathcal{L}\{y''\} = s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - s$

$\mathcal{L}\{\cos t\} = \frac{s}{s^2+1}$

$s^2Y(s) - s - 2(sY(s) - 1) + 2Y(s) = \frac{s}{s^2+1}$

$(s^2-2s+2)Y(s) - s + 2 = \frac{s}{s^2+1}$

$$\boxed{Y(s) = \frac{1}{s^2-2s+2}\left(s - 2 + \frac{s}{s^2+1}\right)}$$