NAME & EID: Solution

M 427K Quiz 7 October 24, 2012 Instructor: James Pascaleff

- Show all work. No books, notes, calculators, or other electronic devices.

This problem is about power series solutions centered at $x_0 = 0$ of the differential equation

$$(1-x)y'' + y = 0 \tag{1}$$

1. (7 points) Seek a power series solution centered at $x_0 = 0$, and find the recurrence relation for the coefficients.

$$y = \sum_{n=0}^{\infty} a_n x^n \qquad y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2}$$

$$(1-x)y'' = y'' - xy'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=0}^{\infty} n(n-1) a_n x^{n-1}$$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0}^{\infty} (n+1) n\, a_{n+1} x^n$$

$$0 = (1-x)y'' + y = \sum_{n=0}^{\infty} \left[(n+2)(n+1) a_{n+2} - (n+1) n\, a_{n+1} + a_n \right] x^n$$

$$\Rightarrow (n+2)(n+1) a_{n+2} - (n+1) n\, a_{n+1} + a_n = 0 \Rightarrow a_{n+2} = \frac{n}{n+2} a_{n+1} - \frac{a_n}{(n+2)(n+1)}$$

2. (3 points) Given the initial conditions $a_0 = 1$, $a_1 = 0$, determine the first four terms of the solution (up to and including the x^3 term).

$$n=0: \quad a_2 = \frac{0}{2} a_1 - \frac{a_0}{2\cdot 1} = -\frac{1}{2}$$

$$n=1: \quad a_3 = \frac{1}{3} a_2 - \frac{a_1}{3\cdot 2} = \frac{1}{3}\cdot\left(-\frac{1}{2}\right) - \frac{0}{6} = -\frac{1}{6}$$

$$y = 1 + 0\cdot x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \cdots = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \cdots$$