NAME & EID: Solutions

M 427K Quiz 6 **October 16, 2012** **Instructor: James Pascaleff**

- Show all work.
- No books, notes, calculators, or other electronic devices.

1. (7 points) Determine the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{3^n} \tag{1}$$

$$L = \lim_{n\to\infty} \left| \frac{(-1)^{n+1}(n+1)^2 (x+2)^{n+1}}{3^{n+1}} \right| \Big/ \left| \frac{(-1)^n n^2 (x+2)^n}{3^n} \right|$$

$$= \lim_{n\to\infty} \frac{(n+1)^2}{n^2} \frac{1}{3} |x+2| = \frac{1}{3}|x+2| \lim_{n\to\infty} \frac{(n+1)^2}{n^2}$$

Now $\lim_{n\to\infty} \frac{(n+1)^2}{n^2} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^2 = 1^2 = 1$, so $L = \frac{1}{3}|x+2|$.

Converges when $L = \frac{1}{3}|x+2| < 1$, meaning $|x+2| < 3$

So radius of convergence $= 3$.

2. (3 points) Rewrite this series as a series whose generic term involves x^n

$$\sum_{n=0}^{\infty} a_n x^{n+2} \tag{2}$$

Answer: $\sum_{n=2}^{\infty} a_{n-2} x^n$

Done Carefully: $\sum_{n=0}^{\infty} a_n x^{n+2} \underset{\boxed{\substack{k=n+2 \\ n=k-2}}}{=} \sum_{k=2}^{\infty} a_{k-2} x^k \underset{\boxed{k\to n}}{=} \sum_{n=2}^{\infty} a_{n-2} x^n$