

- Show all work.
- No books, notes, calculators, or other electronic devices.

This problem is about the second order linear homogeneous ordinary differential equation

$$y'' + y' - 2y = 0 \quad (1)$$

1. (4 points) Write down the characteristic equation and find its solutions.

$$r^2 + r - 2 = 0$$

$$r = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2}$$

$$r_1 = \frac{-1+3}{2} = 1 \quad r_2 = \frac{-1-3}{2} = -2$$

2. (3 points) Write down the general solution of the differential equation.

$$y = c_1 e^t + c_2 e^{-2t}$$

3. (3 points) Solve the initial value problem $y(0) = 1$, $y'(0) = 1$.

$$y' = c_1 e^t - 2c_2 e^{-2t}$$

$$\left\{ \begin{array}{l} y(0) = c_1 + c_2 = 1 \\ y'(0) = c_1 - 2c_2 = 1 \end{array} \right.$$

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$$c_1 = 1 + 2c_2$$

$$(1 + 2c_2) + c_2 = 1$$

$$3c_2 = 0 \rightarrow c_2 = 0$$

$$\text{so } \begin{cases} c_1 = 1 \\ c_2 = 0 \end{cases}$$

$$\boxed{y(t) = e^t}$$