name\&em: SOLUTIONS

M 427K Quiz 3
September 19, 2012

- Show all work.
- No books, notes, calculators, or other electronic devices.

1. (8 points) Consider the autonomous ordinary differential equation

$$
\begin{equation*}
\frac{d y}{d t}=y\left(1-y^{2}\right) \tag{1}
\end{equation*}
$$

Find the critical points, and determine wether each is stable, unstable, or semistable. Hint: it may be helpful to draw a plot of $y\left(1-y^{2}\right)$ versus $y$.

$$
f(y)=y\left(1-y^{2}\right)
$$

$$
\lambda y=0
$$



$$
\begin{gathered}
y 1-y^{2}=0 \quad y y=-1 \\
y=1 \text { stable } \\
+y=0 \text { unstable } \\
y=-1 \text { stable }
\end{gathered}
$$

2. (1 point) How does the solution starting at $y(0)=0.5$ behave as $t \rightarrow \infty$ ?

Start at $y(0)=0.5$ then increase to the stable critical point at $1: y \rightarrow 1$
3. (1 point) How does the solution starting at $y(0)=0$ behave as $t \rightarrow \infty$ ?

0 is a critical point so $y(t)=0$ is
a constant solution $y \rightarrow 0$

