NAME & EID: Solutions

M 427K Quiz 2 **September 12, 2012** **Instructor: James Pascaleff**

- Show all work.
- No books, notes, calculators, or other electronic devices.

1. (5 points) Consider the first order linear ODE

$$\frac{dy}{dt} + 2ty = 2te^{-t^2} \tag{1}$$

Find an integrating factor for this equation, that is, a function $u(t)$ such that

$$\frac{d}{dt}[u(t)y] = u(t)\frac{dy}{dt} + u'(t)y = u(t)\left[\frac{dy}{dt} + 2ty\right] \tag{2}$$

If you remember the formula for $u(t)$, you don't need to rederive it.

$u(t) = e^{\int p(t)dt}$, where $p(t) = 2t$.

$\int 2t\, dt = t^2$

so $u(t) = e^{t^2}$

2. (5 points) Multiply the equation by $u(t)$ and proceed to solve it. Your solution should involve an undetermined constant.

$$e^{t^2}y' + 2te^{t^2}y = 2te^{-t^2}e^{t^2} = 2t$$

$$(e^{t^2}y)' = 2t$$

$$e^{t^2}y = \int 2t\,dt = t^2 + c$$

$$y = e^{-t^2}(t^2 + c)$$