NAME & EID: Solutions

M 427K Quiz 11 **December 5, 2012** **Instructor: James Pascaleff**

- Show all work. No books, notes, calculators, or other electronic devices.

1. (10 points) Find the solution to the heat conduction problem

$$\frac{\partial u}{\partial t} = 100\frac{\partial^2 u}{\partial x^2}$$

On the interval $0 < x < 1$, with boundary conditions $u(0,t) = 0$, $u(1,t) = 0$, and initial temperature distribution

$$u(x,0) = \sin 2\pi x - \sin 5\pi x$$

General solution $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2\alpha^2 t/L^2} \sin \frac{n\pi x}{L}$

Here $L=1$ and $\alpha^2 = 100$

$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2 100t} \sin n\pi x$$

$$u(x,0) = \sum_{n=1}^{\infty} c_n \sin n\pi x \overset{?}{=} \sin 2\pi x - \sin 5\pi x$$

Thus we have $c_2 = 1$, $c_5 = -1$ and all other $c_n = 0$.

so solution is

$$u(x,t) = e^{-2^2\pi^2 100t} \sin 2\pi x - e^{-5^2\pi^2 100t} \sin 5\pi x$$

$$= e^{-400\pi^2 t} \sin 2\pi x - e^{-2500\pi^2 t} \sin 5\pi x.$$