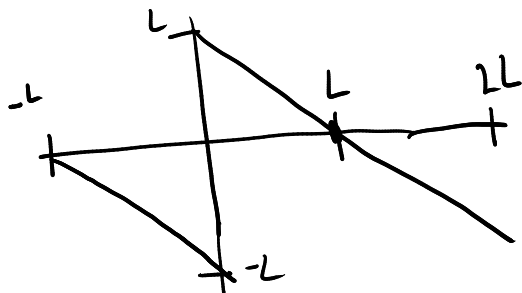


- Show all work. No books, notes, calculators, or other electronic devices.

1. (3 points) If $f(x) = L - x$ on the interval $0 < x < 2L$, and $f(x + 2L) = f(x)$, find a formula for $f(x)$ on the interval $-L < x < 0$.



$$f(x) = -L - x \quad \text{on} \quad -L < x < 0$$

2. (7 points) Consider the function $f(x)$ which is periodic with period $2L$ and determined on the interval $-L \leq x < L$ by

$$f(x) = \begin{cases} 1, & -L \leq x < 0 \\ 0, & 0 \leq x < L \end{cases}$$

Find the coefficient b_1 in the Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_m \cos \frac{m\pi x}{L} + b_m \sin \frac{m\pi x}{L}$$

$$\begin{aligned} b_1 &= \frac{1}{L} \int_{-L}^L f(x) \sin \frac{\pi x}{L} dx = \frac{1}{L} \int_{-L}^0 \sin \frac{\pi x}{L} dx = \frac{1}{L} \left[-\frac{L}{\pi} \cos \frac{\pi x}{L} \right]_{-L}^0 \\ &= \frac{1}{L} \frac{L}{\pi} \left[-\cos 0 + \cos(-\pi) \right] = \frac{1}{\pi} \left[-1 - 1 \right] = -\frac{2}{\pi} \end{aligned}$$