NAME:

EID:

M 427K Exam 1 Version B **October 9, 2012** **Instructor: James Pascaleff**

Problem	Possible	Actual
1	16	
2	20	
3	12	
4	16	
5	16	
6	12	
7	8	
Total	100	

INSTRUCTIONS:

- Do all work on these sheets.
- Show all work.
- No books, notes, calculators, or other electronic devices.

1. AUTONOMOUS EQUATIONS. (16 points) Consider the first order autonomous equation

$$\frac{dy}{dt} = (y-1)(y-2)(y-3)$$

(a) (4 points) Find the critical points of the equation.

(b) (4 points) Draw a plot of $f(y) = (y-1)(y-2)(y-3)$, and use it to determine where solutions are increasing or decreasing.

(c) (4 points) Determine whether each equalibrium is stable, unstable, or semistable.

(d) (4 points) Make a plot of the solutions to the equation.

2. FIRST ORDER SOLUTION TECHNIQUES. (20 points) For parts (a) and (b), Find the general solution to the equation, either by finding a formula for y, or by finding an implicit equation for y.

(a) (8 points) $\frac{dy}{dx} = yx/(1+y^2)$

(b) (8 points) $y' + (2/t)y = t^2 + t^{-2}$ (for $t > 0$)

(c) (4 points) Is the equation $(2x + 4xy) + (2x^2)\frac{dy}{dx} = 0$ exact? Explain how you know, but do not solve it.

3. Complex Exponentials. (12 points)

(a) (6 points) Use Euler's formula to write $e^{5+\pi i} + e^{(\pi/2)i}$ in the form $a + bi$ where a and b are real.

(b) (6 points) Find nonzero constants c_1 and c_2 so that $c_1 e^{4it} + c_2 e^{-4it}$ is real for all t. There are many correct answers.

4. Homogeneous Second Order Linear Equations. (16 points) For each of the following equations, find a pair of solutions that form a fundamental set. You do not need to derive your answer from first principles. If it happens the roots are complex, you should find a pair of real solutions (half credit for complex solutions only).

   (a) (4 points) $y'' - 3y' + 7y = 0$

   (b) (4 points) $y'' + 8y' + 16y = 0$

   (c) (4 points) $y'' - 4y' + 3y = 0$

   (d) (4 points) Find the Wronskian of your solutions to part (c).

5. NONHOMOGENEOUS EQUATIONS. (16 points) Solve the initial value problem

$$y'' + y' - 2y = 10\cos(t), \quad y(0) = -3, \quad y'(0) = 1$$

You may use any methods, but you must show your work.

6. Theory of Linear Equations. (12 points) Consider the second order linear differential operator

$$L[y] = y'' + \sin(t)y' + e^{2t}y$$

Suppose we want to solve the nonhomogeneous equation

$$L[y] = 4\frac{\sin(t)}{t} + 3\ln(t)$$

This is very difficult to do from scratch, but suppose that a very smart colleague finds for us functions Y_1 and Y_2 solving the equations

$$L[Y_1] = \frac{\sin(t)}{t}, \qquad L[Y_2] = 2\ln(t)$$

How can we use Y_1 and Y_2 to get a solution to the original equation?

7. VIBRATIONS. (8 points) Suppose that a damped mass-on-a-spring is described by the equation

$$5u'' + \gamma u' + 10u = 0$$

When γ is small, the solutions oscillate, while when γ is large they do not. Find the value of γ where the cross-over between these two behaviors occurs. In other words, find the value of γ for which the system is critically damped.