Final Review

Course Evaluations end Today

Exam 3 hours Friday May 11 2-5pm
in 6.122

2 sheets of notes permitted

~10 problems

Final is cumulative: test all parts of course

30% about chapters 7 8

Summary

<u>Prelude</u> combinatorics ch 1

permutations (some indistinguishable objects)

combinations & binomials coeffs $\binom{n}{k}$

binomial theorem and <u>combinatorials proofs</u>

prove by counting same thing two different ways.

Discrete Probability Ch 2, 3, 4

2] Sample spaces and events. Axioms of probability

S' sample space E event = subset of S'

$P(E)$ $0 \leq P(E) \leq 1$

$P(S') = 1$

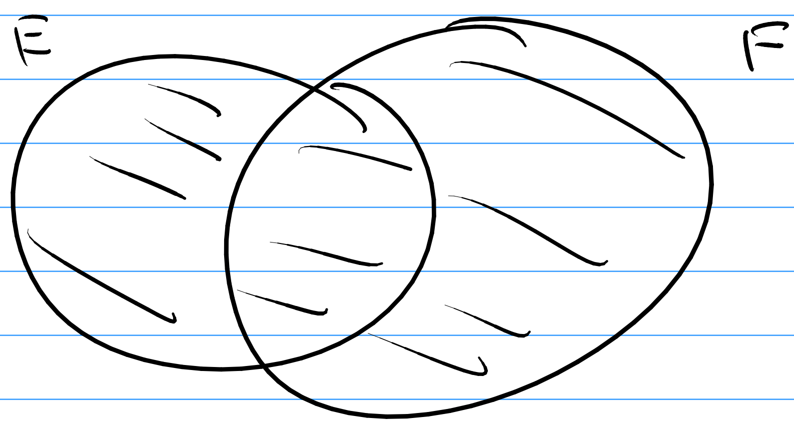
$P(E \cup F) = P(E) + P(F)$
provided that $E \cap F = \emptyset$

Set operations $\cap, \cup$ de Morgan's laws, complement

Venn diagrams

Inclusion-exclusion formula

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$



- Examples of discrete probability coming from situations where all outcomes are equally likely.
  $\Rightarrow$ reduces to combinatorics

3) Conditional probability

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

independent events: $P(EF) = P(E)P(F)$

$$P(E|F) = P(E)$$

Two key lemmas:

Conditioning on a complete set of mutually exclusive events

$H_1, H_2, H_3, \ldots, H_n$

(i) $H_i \cap H_j = \emptyset$ unless $i=j$.

(ii) $H_1 \cup H_2 \cup \cdots \cup H_n = S$

$$P(E) = P(E|H_1)P(H_1) + \cdots + P(E|H_n)P(H_n)$$

Trick: "reversing the order of conditioning"

$$P(F|E) = \frac{P(FE)}{P(E)} = \frac{P(E|F)P(F)}{P(E)}$$

Bayes law:

$$P(H_1 | E) = \frac{P(E|H_1)P(H_1)}{\sum_{i=1}^{n} P(E|H_i)P(H_i)}$$

H_1 = guilty H_2 = not guilty

prior probability

$$P(\text{guilty} | E) = \frac{P(E|\text{guilty})P(\text{guilty})}{P(E|\text{guilty})P(\text{guilty}) + P(E|\text{not guilty})P(\text{not guil}}$$

conditional independence

$$P(E_1 E_2 | F) = P(E_1|F)P(E_2|F)$$

we say E_1 and E_2 are conditionally independent given F.

4] random variables, mass function, expectation & variance

Gambling examples: Expected winnings determine whether its a good idea to play.

Binomial (n,p), Poisson λ, geometric, negative binomial, hypergeometric

Poisson approximation to binomial p small n large, $\lambda = np$

Continuous RVs Ch 5

density function, Cumulative distribution function.

$$P(a<X<b) = \int_a^b f_X(x)\,dx \qquad F_X(a) = \int_{-\infty}^{a} f_X(x)\,dx$$

expectation and variance

memorylessness property

Uniform, normal, exponential random variables

X normal with mean μ, variance σ^2

$\frac{X-\mu}{\sigma}$ normal with mean 0 and variance 1

Theory of RVs 6, 7, 8

6] joint mass / density functions

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

independent RV's.

X, Y independent if $f(x,y) = f_X(x) f_Y(y)$

$$p(x,y) = p_X(x)\,p_Y(y)$$

Sum of independent random variables

$$f_{X+Y}(a) = \int_{-\infty}^{\infty} f_X(x) f_Y(a-x)\,dx$$

7] Expectations of functions of random variables

$$E[g(X,Y)] = \iint g(x,y) f(x,y)\,dx\,dy$$

→ sums products $E[X+Y] = E[X] + E[Y]$

$$\mathrm{Cov}(X,Y) = E[XY] - E[X]E[Y]$$

X, Y independent $\Rightarrow \mathrm{Cov}(X,Y) = 0.$

$\mathrm{Var}(X) = \mathrm{Cov}(X,X)$ ⇒ variances of sums

$$\mathrm{Cov}\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right) = \sum_{i=1}^{n}\sum_{j=1}^{m} \mathrm{cov}(X_i, Y_j)$$

if $X_1, X_2, \ldots, X_n$ are independent

then $\mathrm{Var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \mathrm{Var}(X_i)$

8]: Markov's, Chebyshev's inequality and Weak law of large numbers.

$$P\{|X-\mu| \geq k\} \leq \frac{\sigma^2}{k^2}$$

Central limit theorem

$X_1, \dots, X_n$ independent identically distributed

$$\frac{X_1 + \cdots + X_n - n\mu}{\sigma\sqrt{n}} \xrightarrow{n \to \infty}$$ standard normal distribution with $\mu = 0$, $\sigma = 1$

Weak law

$$P\left\{ \left| \frac{X_1 + \cdots + X_n - n\mu}{n} \right| \geq \varepsilon \right\} \xrightarrow[n \to \infty]{} 0$$

Strong law of large numbers:

$$P\left\{ \lim_{n \to \infty} \frac{X_n + \cdots + X_n}{n} = \mu \right\} = 1$$

Office hours Mon 1-4
Wed 9:30-12

otherwise email.