# starting Chapter 2

HW 2 p. 18-19 : 13, 14

Chapter 2 problems 1, 2, 3, 4, 5

Theoretical exercises 1, 2, 3, 6, 7, 9

## Axioms of Probability

One attempt to define probability

The frequency with which an event occurs.

Take an experiment, run it n times, and let $n(E)$ be the number of times event $E$ happens.

"$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$"

This doesn't work as a definition, but we'll use a different definition, that will allow us to prove a very similar statement.

(The Strong Law of Large Numbers)

# Sample Spaces and Event

Abstractly

Sample space — is a set, the set of possible outcomes of an experiment

Event — is a subset of the sample space

Probability — is a number associated to each event

| Experment | Sample space |
| --- | --- |
| Flipping a coin | $S = \{H, T\}$ (H: heads, T: tails) |
| | S is a set with two elements |
| Flipping a coin twice in a row | $S = \{(H,H), (H,T), (T,H), (T,T)\}$ |
| | 4 elements |
| outcome of a race with 7 competitors | $S = \{$ all 7! permutations of the set of competitors $\}$ |

(set-builder notation)

Time it takes for a Lab rat to finish a maze.

$S = \{t : 0 \le t < \infty\}$

= set of all nonnegative numbers.

5 is in $S$ by $-1$ is not

Events – a subset of the sample space

Subset: $E$ is a subset of $S$, if every element of $E$ is also an element of $S$.

"$x$ is an element of $S$" $\iff$ $x \in S$

$E$ is a subset of $S$

means is an element of.

means (If $x \in E$ then $x \in S$.)

notation $E \subset S$

If we perform the experiment, get outcome $x \in S$.

If $x \in E$, then we say event $E$ has occured.

* Two coin flips $S = \{(H,H), (H,T), (T,H), (T,T)\}$

event $E = \{(H,H), (H,T)\}$

$E$ is a subset of $S$, it represents the event of getting heads on the first flip.

* Race with 7 people $S = \{$all 7! perms$\}$

$E = \{$permutations starting with 5$\}$

is the event that runner number 5 wins.

Q A die is rolled twice.

What is sample space?

$S = \{(1,1), (1,2), (1,3), \ldots$

$(2,1), (2,2), \ldots$

all pairs of numbers between 1 and 6. $\vdots$

$(6,1), (6,2), \ldots (6,6)\}$

Event that the sum of rolls is 7?

$E = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

Event that the sum of the rolls is 12?

$E = \{(6,6)\}$

The notion of probability assigns a number $P(E)$ to each event subject to the following axioms

Axiom 1 $0 \leq P(E) \leq 1$

Axiom 2 $P(S) = 1$

Axiom 3 If $E_1, E_2, \ldots$ is a sequence of events satisfying $E_i \cap E_j = \emptyset$ (for all $i \neq j$)

Then $P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

# Operations on Sets

Fix a sample space $S$ (we never mix sample spaces)

Let $E$ and $F$ be two events ($E \subset S$ and $F \subset S$)

**Union:** $E \cup F$ is the event consisting of outcomes in $E$ or in $F$ or in Both.

$x \in E \cup F \iff x \in E$ or $x \in F$ or both

$E \cup F$ occurs if $E$ occurs or $F$ occurs or both occur.

**Intersection:** $EF$ (also written $E \cap F$) consists of outcomes both in $E$ and in $F$

$x \in EF \iff x \in E$ and $x \in F$

$EF$ occurs if $E$ occurs and $F$ occurs.

Two coin flips

$E = \{(H,H), (H,T)\}$ first flip heads

$F = \{(T,H), (H,H)\}$ second flip heads

$E \cup F = \{(H,H), (H,T), (T,H)\}$

first or second flip heads

$EF = \{(H,H)\}$ both flips heads

Two die rolls

$E$ = sum of rolls is 6

$F$ = sum of rolls is 12

$EF = \{\} = \emptyset$ the empty set

the empty event.